Scattering by small particles

Note Title 3/19/2016

Consider a small defect that is excited by a plane wave,

(8.1) $$\begin{cases} \vec{E}_{inc} = \vec{\mathcal{E}}_0 E_0 e^{-i\omega t + ik\vec{n}_0\cdot\vec{r}} \\ \vec{H}_{inc} = \vec{n}_0 \times \vec{E}_{inc}/Z_0 \end{cases}$$

The scattered light is:

(8.2) $$\begin{cases} \vec{E}_{sc} = \frac{1}{4\pi\epsilon_0} k^2 \frac{e^{ikr}}{r}\left[(\vec{n}\times\vec{p})\times\vec{n} - \vec{n}\times\vec{m}/c\right] \\ \vec{H}_{sc} = (\vec{n}\times\vec{E}_{sc})/Z_0 \end{cases}$$

where $\vec{p}$ and $\vec{m}$ are electric and magnetic dipole moment of the particle.

differential cross-section:

(8.3) $$\frac{d\sigma}{d\Omega}(\vec{n},\vec{\mathcal{E}},\vec{n}_0,\vec{\mathcal{E}}_0) = \frac{r^2\frac{1}{2Z_0}\left|\vec{\mathcal{E}}^*\cdot\vec{E}_{sc}\right|^2}{\frac{1}{2Z_0}\left|\vec{\mathcal{E}}_0^*\cdot\vec{E}_{inc}\right|}$$

$$\left|\vec{\mathcal{E}}^*\cdot\vec{E}_{sc}\right|^2 = \frac{1}{16\pi^2\epsilon_0}\frac{k^4}{r^2}\left|\mathcal{E}^*\cdot\left(\vec{p} - (\vec{p}\cdot\vec{n})\vec{n} - \vec{n}\times\vec{m}/c\right)\right|^2 =$$

$$= \frac{1}{16\pi^2\epsilon_0}\frac{k^4}{r^2}\left|\vec{\mathcal{E}}^*\cdot\vec{p} - \vec{\mathcal{E}}^*\cdot\vec{n}\times\frac{\vec{m}}{c}\right|^2 =$$

$$= \frac{1}{16\pi^2\epsilon_0}\frac{k^4}{r^2}\left|\vec{\mathcal{E}}^*\cdot\vec{p} - \frac{\vec{m}}{c}\cdot\vec{\mathcal{E}}^*\times\vec{n}\right|^2 = \frac{1}{16\pi^2\epsilon_0}\frac{k^4}{r^2}\left|\vec{\mathcal{E}}^*\cdot\vec{p} + \frac{\vec{m}}{c}\cdot(\vec{n}\times\vec{\mathcal{E}}^*)\right|^2$$

(8.4) $$\Rightarrow \boxed{\frac{d\sigma}{d\Omega} = \frac{k^4}{(4\pi\epsilon_0 E_0)^2}\left|\vec{\mathcal{E}}^*\cdot\vec{p} + (\vec{n}\times\vec{\mathcal{E}}^*)\cdot\frac{\vec{m}}{c}\right|^2}$$

Consider now scattering by small particle:

① dielectric particle: ϵ, $r \ll \lambda_0$

The problem can be considered as quasi-static problem:

$$(B5)\quad \begin{cases} V_{out} = \left(\dfrac{A}{r^2} - r\right)\cos\varphi \\[2ex] V_{in} = \sum\limits_m B r \cos\varphi \end{cases}$$

$$@\, R;\quad \begin{cases} \dfrac{A}{R^2} - R = -BR\,; \\[2ex] \left(-\dfrac{2A}{R^3} - 1\right) = -B\epsilon_r \end{cases} \Rightarrow \quad 3 = B(2+\epsilon_r), \quad B = \frac{3}{2+\epsilon_r}$$

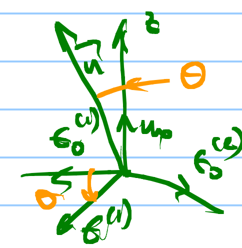
$$E_{in} = E_{out}\frac{3}{2+\epsilon_r}\,; \quad P = (\epsilon - \epsilon_0)E = \epsilon_0(\epsilon_r - 1)E$$

$$p = \frac{4\pi}{3}R^3 \vec{P} = \frac{4\pi R^3}{3} E_{out}\epsilon_0 \frac{3(\epsilon_r - 1)}{\epsilon_r + 2} = 4\pi R^3 E_{out}\epsilon_0 \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$(B6)\quad \boxed{\vec{p} = 4\pi\epsilon_0 \frac{\epsilon_r - 1}{\epsilon_r + 2} R^3 \bar{E}_{out}}\,, \quad \vec{m} = 0$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{k^4}{(4\pi\epsilon_0 E_0)^2}\left|\bar{\epsilon}^* \cdot \vec{E}_0\right| \left(4\pi\epsilon_0 \frac{\epsilon_r - 1}{\epsilon_r + 2} R^3\right)^2 =$$

$$\boxed{\frac{d\sigma}{d\Omega} = k^4 R^6 \left|\frac{\epsilon_r - 1}{\epsilon_r + 2}\right|^2 \left|\bar{\epsilon}^* \cdot \bar{\epsilon}_0\right|^2}$$



For linearly-polarized $\bar{\epsilon}_0$:

$$(B7)\quad \boxed{\begin{aligned} \frac{d\sigma_\parallel}{d\Omega} &= \frac{k^4 R^6}{2}\left|\frac{\epsilon_r - 1}{\epsilon_r + 2}\right|^2 \cos^2\theta \\ \frac{d\sigma_\perp}{d\Omega} &= \frac{k^4 R^6}{2}\left|\frac{\epsilon_r - 1}{\epsilon_r + 2}\right|^2 \end{aligned}}$$

small PEC particle:

$$V_{out} = \left(\frac{A}{r^2} - rE_0\right)\cos\varphi$$

@ R : V = const $\Rightarrow$ $A = -R^3 E_0$

$$V_{scat} = \frac{R^3 E_0}{r^2}\cos\varphi = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cos\theta}{r^2}$$

(8.8) $\Rightarrow$ $\boxed{\vec{p} = 4\pi\epsilon_0 R^3 E_0}$

PEC sphere has magnetic moment! $(\vec{B}_n = 0)$

when $\vec{j} = 0$, $\vec{H} = -\nabla\varphi_m$; $\vec{B} = \mu\vec{H}$; $H \Leftrightarrow E$, $p \Leftrightarrow M$

(8.8b) $\Rightarrow$ $\vec{m} = 4\pi R^3 \vec{H}_{out}\,\frac{\mu_r - 1}{\mu_r + 2} = \boxed{-2\pi R^3 \vec{H}_{out}}$

$$\frac{d\sigma}{d\Omega} = \frac{k^4}{(4\pi\epsilon_0 E_0)^2}\left|\vec{\epsilon}^*\cdot 4\pi\epsilon_0 R^3 \vec{E}_0 + \frac{2\pi}{c}\left(\vec{\epsilon}^*\times\vec{n}\right)\cdot R^3\vec{H}_0\right|^2 =$$

$$= \frac{k^4R^6}{(2\epsilon_0)^2}\left|\vec{\epsilon}^*\cdot\vec{E}_0\cdot 2\epsilon_0 - (\vec{n}\times\vec{\epsilon}^*)\cdot(\vec{n}_0\times\vec{E}_0)\frac{1}{\mathcal{Z}c}\right|^2 =$$

$$= k^4R^6\left|\vec{\epsilon}^*\cdot\vec{\epsilon}_0 - (\vec{n}\times\vec{\epsilon}^*)\cdot(\vec{n}_0\times\vec{\epsilon}_0)\frac{1}{2}\right|^2$$

(8.9)
$$\boxed{\begin{aligned}\frac{d\sigma_\parallel}{d\Omega} &= \frac{k^4R^6}{2}\left|\cos\theta - \frac{1}{2}\right|^2\\ \frac{d\sigma_\perp}{d\Omega} &= \frac{k^4R^6}{2}\left|1 - \frac{1}{2}\cos\theta\right|^2\end{aligned}}$$