Metals

Note Title 2/9/2016

We have previously derived permittivity of metal (3.116)

$$\epsilon = \epsilon_0 - \frac{e^2 N}{m}\frac{1}{\omega^2 + i\gamma_0\omega} = \epsilon_0 - \frac{e^2 N}{m\omega}\frac{\omega - i\gamma_0}{\omega^2+\gamma_0^2} =$$

$$= \epsilon_0 - \frac{e^2 N}{m(\omega^2+\gamma_0^2)} + i\frac{e^2 N \gamma_0}{m\,\omega(\omega^2+\gamma_0^2)} \Rightarrow$$

(5.1) $$\boxed{\epsilon = \epsilon_0\left[1 - \frac{\omega_p^2}{\omega^2+\gamma^2} + i\frac{\omega_p^2\gamma}{\omega(\omega^2+\gamma_0^2)}\right]}$$

with $\boxed{\omega_p^2 = \frac{e^2 N}{m\epsilon_0}}$ – plasma freq (squared)

The limits are of practical interest:

① $\gamma_0 \ll \omega \lesssim \omega_p$ [visible ... IR for noble metals]

② $\omega \sim 0$ $(\omega \ll \gamma_0)$ [THz & below for N.M.]

(5.2a) limit ①: $$\epsilon \simeq \epsilon_0\left[1 - \frac{\omega_p^2}{\omega^2} + i\frac{\gamma\,\omega_p^2}{\omega^3}\right] \simeq$$

$$\simeq \epsilon_0\frac{\omega_p^2}{\omega^2}\left[-1 + i\frac{\gamma}{\omega}\right]$$

permittivity is (large) negative with small imag part

(5.2b) limit ②: $$\epsilon \simeq \epsilon_0\left[1 - \frac{\omega_p^2}{\gamma^2} + i\frac{\omega_p^2\gamma_0}{\omega\gamma_0^2}\right] \simeq \epsilon_0\frac{\omega_p^2}{\omega\gamma_0}i$$

σ_{dc}

$\sigma = -i\omega\epsilon_0\epsilon_r$

permittivity is large and imaginary

In both cases, waves do not propagate inside the metal.

In fact, waves that do penetrate the metal have the dependence:

(5.3) $$H, E \propto e^{ik_z z}, \text{ with } k_z = \sqrt{\epsilon_r \frac{\omega^2}{c^2} - k_x^2} \simeq \sqrt{\epsilon_r}\,\frac{\omega}{c} \gg k_x$$

in limit ① $k_z \simeq \sqrt{-\frac{\omega_p^2}{\omega^2}}\,\frac{\omega}{c} \simeq \frac{\omega_p}{c}\, i = \frac{i}{\delta}$ $\left[H, E \propto e^{-\frac{z}{\delta}} \right]$

in limit ② $k_z \simeq \sqrt{i\frac{\omega_p^2 \omega}{\gamma_0 c^2}} \simeq \underbrace{\frac{\omega_p}{c}\sqrt{\frac{\omega}{\gamma_0}}}_{\frac{1}{\delta}}\,\frac{1+i}{\sqrt{2}}$ $\left[H, E \propto e^{-\frac{z}{\delta}(1+i)} \right]$

$|H, E| \propto e^{-z/\delta}$

Note that in both cases the amplitudes of E & H are related to each other via

$$k \times H_0 = -\epsilon_0 \epsilon \omega E_0$$

Consider, for now a TM-polarized wave:

$$E_0 = \begin{bmatrix} E_{0x} \\ 0 \\ E_{0z} \end{bmatrix} ; \quad H_0 = \begin{bmatrix} 0 \\ H_0 \\ 0 \end{bmatrix}$$

$$E_0 = -\frac{1}{\epsilon_0 \epsilon \omega} k \times H = -\frac{1}{\epsilon_0 \epsilon \omega} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ k_x & 0 & k_z \\ 0 & H_0 & 0 \end{vmatrix} = +\frac{H_0}{\epsilon_0 \epsilon_r \omega} \begin{bmatrix} k_z \\ 0 \\ -k_x \end{bmatrix} \simeq \frac{H_0 c}{\epsilon_0 \sqrt{\epsilon_r}} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

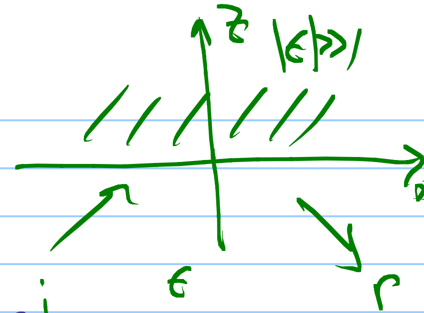
$$= H_0 \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

normal into metal

(5.4) $$\boxed{\vec{E}_\tau = \left[\vec{H}_{0\tau} \times \vec{n}\right] \cdot \zeta}$$ ← the relationship is valid for TE waves as well.

$$\boxed{\zeta = \sqrt{\frac{\mu}{\epsilon}}}$$ - surface impedance $\ll \zeta_0 \simeq 376\,\Omega$

Eq (5.4) can be used as a boundary condition to calculate reflection:
(for simplicity, use isotropic medium, TM wave)



(5.5) $$E_{inc} = \frac{H_i}{\epsilon_0 \epsilon \omega} \begin{bmatrix} k_z \\ 0 \\ -k_x \end{bmatrix} e^{-i\omega t + i\bar{k}\cdot\bar{r}}$$

$$E_r = \frac{H_r}{\epsilon_0 \epsilon \omega} \begin{bmatrix} -k_z \\ 0 \\ -k_x \end{bmatrix} e^{-i\omega t + i k_x x - i k_z z}$$

$$E(z=0) = e^{-i\omega t + i k_x x} \frac{1}{\epsilon_0 \epsilon \omega} \begin{bmatrix} k_z(H_i - H_r) \\ 0 \\ k_x(H_i + H_r) \end{bmatrix}$$

$$H(z=0) = e^{-i\omega t + i k_x x} \begin{bmatrix} 0 \\ H_i + H_r \\ 0 \end{bmatrix}$$

$$(5.4) \Rightarrow \frac{1}{\epsilon_0 \epsilon \omega} k_z (H_i - H_r) = (H_i + H_r)\zeta$$

(5.6)

$$r = \frac{H_r}{H_i} = \frac{k_z - \zeta \epsilon_0 \epsilon \omega}{k_z + \zeta \epsilon_0 \epsilon \omega} = \frac{k_z - \frac{\omega}{c}\sqrt{\frac{\epsilon}{\epsilon_m}}}{k_z - \frac{\omega}{c}\sqrt{\frac{\epsilon}{\epsilon_m}}}$$

$$r = \frac{\sqrt{\epsilon_m}\cos\theta - \sqrt{\epsilon}}{\sqrt{\epsilon_m}\cos\theta + \sqrt{\epsilon}}$$

$$R = |r|^2 = \left| \frac{k_z - \zeta \epsilon_0 \epsilon \omega}{k_z + \zeta \epsilon_0 \epsilon \omega} \right|^2.$$

For reflective materials, $R - 1 = A$ - absorption