Differential Equations
Homework 7 (Revised)
Due Mar. 20, 2024, 9:59 am

Note:

- Please show all of your work (writing a list of answers is not sufficient).

- Please indicate the people you worked with.

- **Please staple your HW.**

- Several random problems will be graded (1 point each).

1. Determine and justify whether the pairs of functions are linearly independent or linearly dependent on the real line
$$f(x) = \pi, \quad g(x) = \cos^2 x + \sin^2 x$$

2. Determine and justify whether the pairs of functions are linearly independent or linearly dependent on the real line
$$f(x) = 1 + x, \quad g(x) = 1 + |x|$$

3. Determine and justify whether the pairs of functions are linearly independent or linearly dependent on the real line
$$f(x) = \sin^2 x, \quad g(x) = 1 - \cos 2x$$

4. Find general solutions of the differential equations

   (a)
   $$y'' - 3y' + 2y = 0$$

   (b)
   $$y'' + 5y' = 0$$

   (c)
   $$2y'' - y' - y = 0$$

5. Find a differential equation with a given general solution

   (a)
   $$y = c_1 e^{10x} + c_2 e^{-10x}$$

   (b)
   $$y = c_1 e^{10x} + c_2 e^{100x}$$