Differential Equations
Homework 5
Due Feb. 28, 2024, 9:59 am

Note:

- Please show all of your work (writing a list of answers is not sufficient).

- Please indicate the people you worked with.

- **Please staple your HW.**

- Several random problems will be graded (1 point each).

1. Sketch (by hands) slope field of
$$\frac{dy}{dt} = y$$

2. Sketch (by hands) slope field of

$$\frac{dp}{dt} = p(10 - p)$$

3. Separate variables and use partial fraction to solve the initial value problem

$$\frac{dx}{dt} = x - x^2, x(0) = 2$$

4. Separate variables and use partial fraction to solve the initial value problem

$$\frac{dx}{dt} = 3x(5 - x), x(0) = 8$$

5. Separate variables and use partial fraction to solve the initial value problem

$$\frac{dx}{dt} = 3x(x - 5), x(0) = 2$$

6. Suppose that the fish population $P(t)$ in a lake is attacked by a disease at time $t = 0$, with the result that the fish cease to produce (so that the birth rate is $\beta = 0$) and the death rate δ (deaths per week per fish) is $k\frac{1}{\sqrt{P}}$. If there were initially 900 fish ($P(0) = 900$) in the lake and 441 were left after 6 weeks ($P(6) = 441$), how long did it take all the fish in the lake to die?

7. Suppose that the number of alligator $x(t)$ (with t in months) of alligators in a swamp satisfies the differential equation

$$\frac{dx}{dt} = 0.0001x^2 - 0.01x$$

(a) If initially there are 25 alligators in the swamp, solve the differential equation to determine what happens to the alligator population in the long run $(t \to \infty)$

(b) Repeat part (a), except with 150 alligators initially.