

Differential Equations

Homework 3

Due Feb. 7, 2024 (Wed.)

Note:

- Please show all of your work (writing a list of answers is not sufficient).
- Please indicate the people you worked with.
- **Please staple your HW.**
- Several random problems will be graded (1 point each).

1. Find the general solution

$$y' + 3y = 2xe^{-3x}$$

2. Find the particular solution of the initial value problem

$$xy' - y = x, \quad y(1) = 7$$

3. Find the particular solution of the initial value problem

$$y' + y = e^x, \quad y(0) = 1$$

4. Find the particular solution of the initial value problem

$$(1 + x)y' + y = \cos(x), \quad y(0) = 1$$

5. Find the particular solution of the initial value problem

$$y' = 1 + x + y + xy, \quad y(0) = 0$$

6. Find the particular solution of the initial value problem

$$(x^2 + 4)y' + 3xy = x, \quad y(0) = 1$$

7. A tank contains 1000 liters (L) of a solution consisting of 100 kg of salt dissolved in water. Pure water is pumped into the tank at the rate of 5 L/s, and the mixture – kept uniform by stirring – is pumped out at the same rate. How long will it be until only 10 kg of salt remains in the tank?

8. A tank initially contains 60 gal of pure water. Brine consisting of 1 lb of salt per gallon enters the tank at 2 gal/min, and the (perfectly mixed) solution leaves the tank at 3 gal/min: thus the tank is empty after exactly 1 h.

- (a) Find the amount of salt in the tank after t minutes.
- (b) What is the maximum amount of salt ever in the tank?