

Differential Equations
Homework 2
Due Jan. 31, 2024 (Wed.)

Note:

- Please show all of your work (writing a list of answers is not sufficient).
- Please indicate the people you worked with.
- Several random problems will be graded (1 point each).

1. Find a function $y = f(x)$ satisfying

$$\frac{dy}{dx} = (x - 2)^2; y(2) = 1$$

2. Find a function $y = f(x)$ satisfying

$$\frac{dy}{dx} = x\sqrt{x^2 + 9}; y(-4) = 0$$

3. Find a function $y = f(x)$ satisfying

$$\frac{dy}{dx} = xe^{-x}; y(0) = 1$$

4. A diesel car gradually speeds up so that for the first 10 s its acceleration is given by

$$\frac{dv}{dt} = 0.12t^2 + 0.6t \quad (ft/s^2).$$

If the car starts from rest ($x_0 = 0$, $v_0 = 0$), find the distance it has traveled at the end of the first 10 s and its velocity at that time

5. Find the general solution of

$$\frac{dy}{dx} + 2xy = 0$$

6. Find the general solution of

$$(1 + x)^2 \frac{dy}{dx} = (1 + y)^2$$

7. Find the general solution of

$$y^3 \frac{dy}{dx} = (y^4 + 1) \cos x$$

8. Find explicit particular solution of

$$\frac{dy}{dx} = 2xy^2 + 3x^2y^2, y(1) = -1$$

9. In a certain culture of bacteria, the number of bacteria increased sixfold in 10 h. How long did it take for the population to double?