Differential Equations
Homework 1
Due Jan. 24, 2024 (Wed.)

Note:

- Please show all of your work (writing a list of answers is not sufficient).
- Please indicate the people you worked with.
- Several random problems will be graded (1 point each).

1. Verify by substitution that $y = 3e^{-2x}$ is a solution of

$$y' + 2y = 0; y = 3e^{-2x}$$

$$y' = -6e^{-2x}$$

$$2y = 3(2e^{-2x})$$

$$y' + 2y = -6e^{-2x} + 2(3e^{-2x}) = 0$$

Therefore,

$$y = 3e^{-2x}$$

is a solution

2. Verify by substitution that $y_1 = \cos 2x$ and $y2 = \sin 2x$ are solutions of

$$y'' + 4y = 0; y_1 = \cos 2x, y2 = \sin 2x$$

$$y_1' = -2\sin 2x, y_1'' = -4\cos 2x$$

$$y_1'' + 4y_1 = -4\cos 2x + 4(\cos 2x) = 0$$

$$y_2' = 2\cos 2x, y_2'' = -4\sin 2x$$

$$y_2'' + 4y_2 = -4\sin 2x + 4(\sin 2x) = 0$$

Therefore,

$$y_1 \text{ and } y_2$$

are solutions of differential equation

3. Verify by substitution that $y_1 = \cos x - \cos 2x$, $y_2 = \sin x - \cos 2x$ are solutions of

$$y'' + y = 3\cos 2x; y_1 = \cos x - \cos 2x, y_2 = \sin x - \cos 2x$$
$$y_1' = -\sin x + 2\sin 2x, y_1'' = -\cos x + 4\cos 2x$$
$$y_1'' + y_1 = (-\cos x + 4\cos 2x) + (\cos x - \cos 2x) = 3\cos 2x$$
$$y_2' = \cos x + 2\sin 2x, y_2'' = -\sin x + 4\cos 2x$$
$$y_2'' + y_2 = (-\sin x + 4\cos 2x) + (\sin x - \cos 2x) = 3\cos 2x$$

Therefore,

$$y_1 \text{ and } y_2$$

are solutions of differential equation

4. Substitute $y = e^{rx}$ into the differential equations to determine all values of the constant r for which $y = e^{rx}$ is a solution of

$$4y'' = y$$

$$4y'' = y; y = e^{rx}$$
$$y' = re^{rx}, y'' = r^2 e^{rx}$$
$$4y'' = y \to 4r^2 e^{rx} = e^{rx} \to 4r^2 = 1 \to r = \pm\frac{1}{2}$$

Therefore,

$$y_1 = e^{\frac{1}{2}x} \text{ and } y_2 = e^{-\frac{1}{2}x}$$

are solutions.

5. Substitute $y = e^{rx}$ into the differential equations to determine all values of the constant r for which $y = e^{rx}$ is a solution of

$$y'' + y' - 2y = 0$$

$$y'' + y' - 2y = 0; y = e^{rx}$$
$$y' = re^{rx}, y'' = r^2 e^{rx}$$
$$y'' + y' - 2y = 0 \to r^2 e^{rx} + re^{rx} - 2e^{rx} = 0 \to r^2 + r - 2 = 0$$
$$\to (r+2)(r-1) = 0 \to r = -2, r = 1$$

Therefore,

$$y_1 = e^{-2x} \text{ and } y_2 = e^{x}$$

are solutions.

6. Verify by substitution that $y(x) = Ce^{-x}$ is a solutions of

$$y' + y = 0$$

and find C using the initial condition $y(0) = 2$

$$y' + y = 0; y(x) = Ce^{-x}, y(0) = 2$$
$$y'(x) = -Ce^{-x}$$
$$y' + y = 0 \rightarrow -Ce^{-x} + (Ce^{-x}) = 0$$

Therefore,

$$y(x) = Ce^{-x}$$

is a solution of differential equation.

$$y(0) = Ce^{0} = 2 \rightarrow C = 2$$

$$y(x) = 2e^{-x}$$

7. Verify by substitution that $y(x) = Ce^{-x} + x - 1$ is a solutions of

$$y' = x - y$$

and find C using the initial condition $y(0) = 10$

$$y' = x - y; y(x) = Ce^{-x} + x - 1, y(0) = 10$$

$$y'(x) = -Ce^{-x} + 1$$

$$y' = x - y \rightarrow -Ce^{-x} + 1 = x - (Ce^{-x} + x - 1) = -Ce^{-x} + 1$$

Therefore,

$$y(x) = Ce^{-x} + x - 1$$

is a solution of differential equation.

$$y(0) = Ce^{-0} + 0 - 1 = 10 \rightarrow C = 11$$

$$y(x) = 11e^{-x} + x - 1$$

8. Write a differential equation of the situation described: The acceleration $\frac{dv}{dt}$ of a vehicle is proportional to the difference between 250 km/h and the velocity of the car

$$\frac{dv}{dt} = k(250 - v)$$

9. Determine by inspection at least one solution of the differential equation. That is, use your knowledge of derivatives to make an intelligent guess. Then verify your guess by substituting it to the differential equation

$$y' = y$$

$$y = e^x$$
$$y' = e^x$$
$$y' = y = e^x$$

10. Determine by inspection at least one solution of the differential equation. That is, use your knowledge of derivatives to make an intelligent guess. Then verify your guess by substituting it to the differential equation

$$y'' + y = 0$$

$$y = \cos x$$
$$y' = -\sin x$$
$$y'' = -\cos x$$
$$(-\cos x) + (\cos x) = 0$$