

Section 5.3

After viewing the lecture videos and reading the textbook, you should be able to answer the following questions:

1. The **definite integral**, $\int_a^b f(x) dx$, of a function f over a closed interval $[a, b]$ is defined to be

$$\int_a^b f(x) dx = \lim_{\|P\| \rightarrow 0} \sum_{k=1}^n f(c_k) \cdot \Delta x_k$$

where P is a partition of $[a, b]$, provided the limit exists.

- What is the integral sign?
 - What is the lower limit of integration?
 - What is the upper limit of integration?
 - What is the integrand?
 - What does dx represent?
2. The properties of definite integrals are:

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^a f(x) dx = 0$$

$$\int_a^b k \cdot f(x) dx = k \cdot \int_a^b f(x) dx$$

$$\int_a^b (f(x) \pm g(x)) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

$$\min f \cdot (b - a) \leq \int_a^b f(x) dx \leq \max f \cdot (b - a)$$

$$f(x) \geq g(x) \text{ on } [a, b] \Rightarrow \int_a^b f(x) dx \geq \int_a^b g(x) dx$$

$$f(x) \geq 0 \text{ on } [a, b] \Rightarrow \int_a^b f(x) dx \geq 0$$

Suppose $\int_1^{13} f(x) dx = 3$, $\int_1^5 f(x) dx = 17$ and $\int_1^{13} g(x) dx = 5$. Find the values of:

- $\int_{13}^1 f(x) dx$
- $\int_{11}^{11} f(x) dx$
- $\int_1^{13} 7 \cdot f(x) dx$
- $\int_1^{13} (f(x) + g(x)) dx$
- $\int_1^{13} (f(x) - g(x)) dx$
- $\int_5^{13} f(x) dx$

3. You were given the following formulas to calculate some simple definite integrals:

$$\int_a^b c dx = c \cdot (b - a), \quad c \text{ any constant}$$

$$\int_a^b x dx = \frac{b^2}{2} - \frac{a^2}{2}, \quad a < b$$

$$\int_a^b x^2 dx = \frac{b^3}{3} - \frac{a^3}{3}, \quad a < b$$

Evaluate the following definite integrals:

- $\int_5^{100} 2 dx$
- $\int_2^4 x dx$
- $\int_3^9 x^2 dx$

4. Which one of the following expressions is not the same as the others:

- $\int_a^b 1 dx$
- $\int_a^b dx$
- $b - a$
- 1

5. What is the average value of a continuous function, $y = f(x)$, over a closed interval, $[a, b]$?