Name ____Section ________________

Student ID Number __________________ Instructor _______________________________

Instructions: No note or calculators are allowed. Answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. **Work must be neat, organized and easily interpreted.**

A page of useful information has been included on the last page of the exam.

Please circle your final answers.

Notes:
$0! = 1$ and if $n > 0$ then $n! = 1 \times 2 \times 3 \times \cdots \times n$

Page #	Point Value	Grade
2	12	
3	12	
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5	6	
6	6	
7	6	
8	6	
9	12	
10	12	
11	10	
12	12	
Total	100	

1. Evaluate the integrals:
   a. (6 pts) $\int_0^2 \; 2x(x^2-1)^3\,dx$

   b. (6 pts) $\int x\ln\sqrt{x}\,dx$

c. (6 pts) $\int_0^{\pi/4} \cos^3(2x)\,dx$

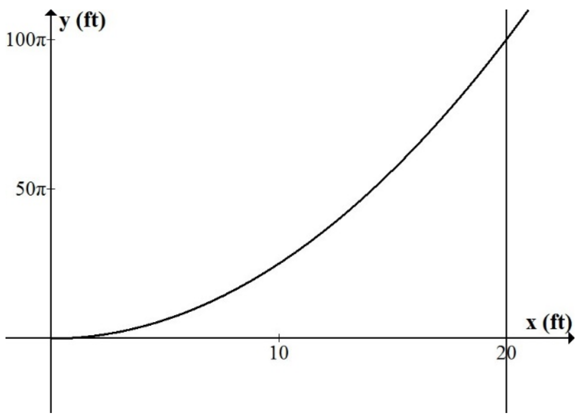
d. (6 pts) $\int \tan(x)\sec^3(x)\,dx$

e. (6 pts) $\int \frac{1}{x^2\sqrt{25-x^2}}\,dx$

f. (6 pts) $\int_0^1 \frac{1}{(x-1)^2}\,dx$ (**Hint**: This is an improper integral.)

g. (6 pts) $\int \frac{x^2-4}{x^3+4x}\,dx$

2. (6 pts) A nose cone for a space reentry vehicle has the shape of the solid that is generated by revolving the region bounded by $y = \frac{1}{4}\pi x^2$, $y = 0$, and $x = 20$ about the x-axis. Find the volume (in ft^3) of the nose cone.



3. (6 pts) Find the arc length of the curve $y = \frac{1}{3}x^{3/2}$ from $\left(5, \frac{25\sqrt{5}}{3}\right)$ to $\left(12, 8\sqrt{3}\right)$.

4. Determine whether the following series **converge** or **diverge**. Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

   a. (6 pts) $\sum_{k=1}^{\infty} \frac{7^k(k+1)}{(2k)!}$

   b. (6 pts) $\sum_{n=2}^{\infty} \frac{n}{(n-1)2^n}$

c. (6 pts) $\sum_{n=1}^{\infty} \frac{e^n}{e^n+n}$

d. (6 pts) $\sum_{k=2}^{\infty} (-1)^k \frac{4}{(\ln k)^2}$

5. (10 pts) Find the interval of convergence and the radius of convergence for the power series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-3)^n}{n}$$

Also, for which values of x does the series converge absolutely? Conditionally?

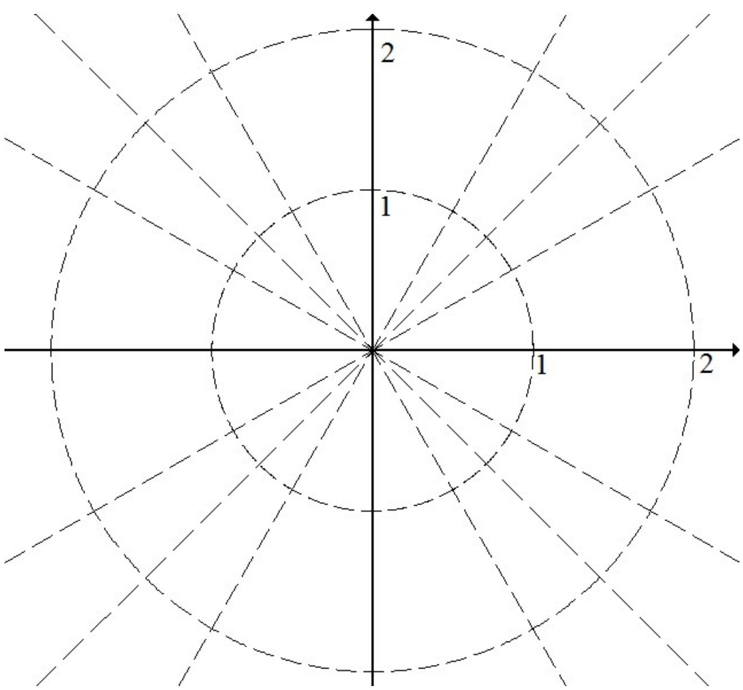
Interval of Convergence: ____________________ Radius of Convergence: __________________

Absolute Convergence: _______________________ Conditional Convergence: ________________

6. (6 pts) Find the Taylor **series** generated by $f(x) = x^3$ at $a = -2$.

7. (6 pts) Sketch the polar curve $r = 2\cos 2\theta$, $0 \le \theta \le \pi$. Clearly label the points where

$$\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \text{ and } \pi$$



Disk Method: $V = \int_a^b \pi[R(x)]^2\,dx$

Washer Method: $V = \int_a^b \pi([R(x)]^2 - [r(x)]^2)\,dx$

Shell Method: $V = \int_a^b 2\pi r(x)h(x)\,dx$

Surface Area: $S = \int_a^b 2\pi \cdot f(x)\sqrt{1 + \left(f'(x)\right)^2}\,dx$

Arc Length Formula: $L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$

Useful Trigonometric Identities: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; $\sin 2\theta = 2\sin\theta\cos\theta$

The n-th Term Test for Divergence: $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

The Limit Comparison Test: Let $\sum a_n$ and $\sum b_n$ be series with positive terms and suppose $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.

a) If L is finite and $L > 0$, then the series both converge or both diverge.

b) If $L = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

c) If $L = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Ratio Test: Let $\sum a_n$ be a series with nonzero terms and suppose $\rho = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$.

a) If $\rho < 1$, the series converges absolutely.

b) If $\rho > 1$ or $\rho = \infty$, the series diverges.

c) If $\rho = 1$, then the test is inconclusive, use a different test.

Alternating Series Test: An alternating series $\sum_{n=1}^{\infty}(-1)^n u_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} u_n$ converges if the following three conditions are satisfied:

1) $u_n > 0$ for all $n \geq N$ 2) $\lim_{n\to\infty} u_n = 0$ 3) $u_n \geq u_{n+1}$ for all $n \geq N$ for some N

The **n-th Taylor polynomial** for f about $x = a$ is $p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

The **Taylor series** for f about $x = a$ is $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

Symmetry Tests for Polar Graphs

1. *Symmetry about the x-axis:* If the point (r,θ) lies on the graph, then $(r,-\theta)$ or $(-r,\pi-\theta)$ also lies on the graph.
2. *Symmetry about the y-axis:* If the point (r,θ) lies on the graph, then $(r,\pi-\theta)$ or $(-r,-\theta)$ also lies on the graph.
3. *Symmetry about the origin:* If the point (r,θ) lies on the graph, then $(-r,\theta)$ or $(r,\theta+\pi)$ also lies on the graph.

Slope of a Polar Curve $r = f(\theta)$ in the Cartesian Plane: $\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$