Name ____ Section ________________

Student ID Number ___________________ Instructor ________________________________

Instructions: No note or calculators are allowed. Answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. **Work must be neat, organized and easily interpreted.**

A page of useful information has been included on the last page of the exam.

Please circle your final answers.

Notes:
$0! = 1$ and if $n > 0$ then $n! = 1 \times 2 \times 3 \times \cdots \times n$

Page #	**Point Value**	**Grade**
3	12	
4	6	
5	7	
6	7	
7	12	
8	6	
9	16	
10	10	
11	14	
12	10	
Total	100	

1. Evaluate the integrals:
   a. (6 pts) $\int_1^4 \frac{1}{\sqrt{x}\,\left(1+\sqrt{x}\right)^2}\,dx$

   b. (6 pts) $\int x^2 \sin(x)\,dx$

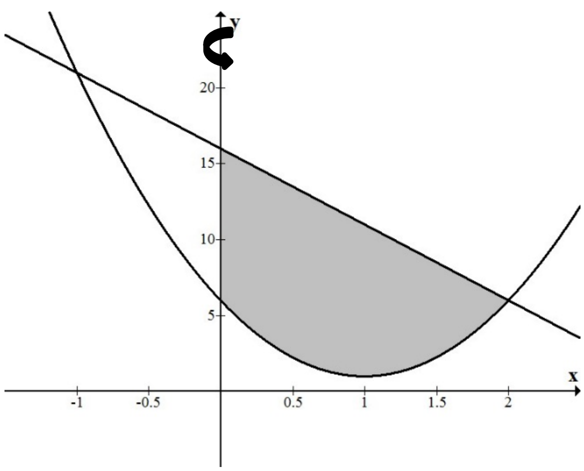
c. (6 pts) $\int_{-2}^{6} \sin^3(\pi x)\cos^2(\pi x)\,dx$

d. (7 pts) $\int_2^4 \frac{\sqrt{x^2-4}}{x} dx$ (Hint: use trigonometric substitution)

e. (7 pts) $\int \frac{3-x}{(3x+1)(x-1)} dx$

f. (6 pts) $\int_0^\infty e^{-2x}\,dx$

2. (6 pts) Set up an integral that finds the volume of the solid generated by revolving the region in the first quadrant bound by $y = 5x^2 - 10x + 6$ and $y = -5x + 16$ about the y-axis. **Do not solve the integral.**



3. (6 pts) Find the area of the surface formed by revolving the graph of $f(x) = x^3$ on the interval $[0,1]$ about the x-axis.

4. Determine whether the following series **converge** or **diverge**. Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

   a. (8 pts) $\sum_{k=1}^{\infty} \frac{n+1}{\sqrt{n^3+2}}$

   b. (8 pts) $\sum_{n=1}^{\infty} \frac{7^n}{n!}$

5. (10 pts) Find the interval of convergence and the radius of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(3x+1)^{n+1}}{2n+2}$$

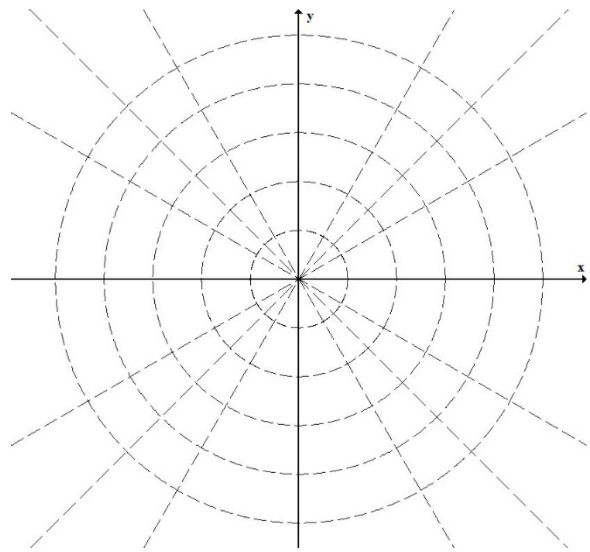
Interval of Convergence: ___________________ Radius of Convergence: _________________

6. (6 pts) Does the following series converge or diverge? If it converges, find its sum.

$$\sum_{n=1}^{\infty} \frac{2^n}{3^{n-1}}$$

7. (8 pts) Find the first four nonzero terms of the Taylor **series** generated by $f(x) = \frac{1}{1-x}$ at $a = 2$.

8. (6 pts) Find the Cartesian coordinates of the following points given in polar coordinates and sketch them on the given coordinate system. Label the points in polar coordinates.



   a. $(r, \theta) = (-3, \pi)$

   b. $(r, \theta) = \left(5, \frac{4\pi}{3}\right)$

9. (4 pts) Find the polar coordinates, $0 \leq \theta < 2\pi$ and $r \geq 0$, of the following points given in Cartesian coordinates.

   a. $(x, y) = \left(\sqrt{2}, \sqrt{2}\right)$

   b. $(x, y) = \left(-4\sqrt{3}, 4\right)$

Disk Method: $V = \int_a^b \pi[R(x)]^2\,dx$

Washer Method: $V = \int_a^b \pi([R(x)]^2 - [r(x)]^2)\,dx$

Shell Method: $V = \int_a^b 2\pi r(x)h(x)\,dx$

Surface Area: $S = \int_a^b 2\pi \cdot f(x)\sqrt{1 + \left(f'(x)\right)^2}\,dx$

Arc Length Formula: $L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$

Useful Trigonometric Identities: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; $\sin 2\theta = 2\sin\theta\cos\theta$

The n-th Term Test for Divergence: $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

The Limit Comparison Test: Let $\sum a_n$ and $\sum b_n$ be series with positive terms and suppose $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.

a) If L is finite and $L > 0$, then the series both converge or both diverge.

b) If $L = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

c) If $L = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Ratio Test: Let $\sum a_n$ be a series with nonzero terms and suppose $\rho = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$.

a) If $\rho < 1$, the series converges absolutely.

b) If $\rho > 1$ or $\rho = \infty$, the series diverges.

c) If $\rho = 1$, then the test is inconclusive, use a different test.

Alternating Series Test: An alternating series $\sum_{n=1}^{\infty}(-1)^n u_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} u_n$ converges if the following three conditions are satisfied:

1) $u_n > 0$ for all $n \geq N$ 2) $\lim_{n\to\infty} u_n = 0$ 3) $u_n \geq u_{n+1}$ for all $n \geq N$ for some N

The **n-th Taylor polynomial** for f about $x = a$ is $p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

The **Taylor series** for f about $x = a$ is $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

Symmetry Tests for Polar Graphs

1. *Symmetry about the x-axis:* If the point (r, θ) lies on the graph, then $(r, -\theta)$ or $(-r, \pi - \theta)$ also lies on the graph.
2. *Symmetry about the y-axis:* If the point (r, θ) lies on the graph, then $(r, \pi - \theta)$ or $(-r, -\theta)$ also lies on the graph.
3. *Symmetry about the origin:* If the point (r, θ) lies on the graph, then $(-r, \theta)$ or $(r, \theta + \pi)$ also lies on the graph.

Slope of a Polar Curve $r = f(\theta)$ in the Cartesian Plane: $\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$