Area and Estimating with Finite Sums

Part 3

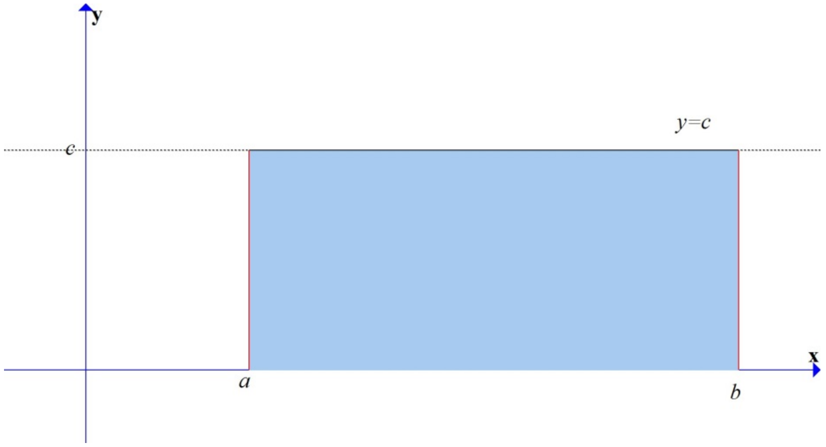
Average Value of a Nonnegative Continuous Function

Average Value

- The average value of $\{x_1, x_2, x_3, \cdots, x_n\}$ is $\frac{x_1+x_2+x_3+\cdots+x_n}{n}$
- What is the average value of a continuous function f on an interval $[a, b]$?

Average Value

- If f is a constant value $c > 0$ on an interval $[a, b]$, then its average value should be c.

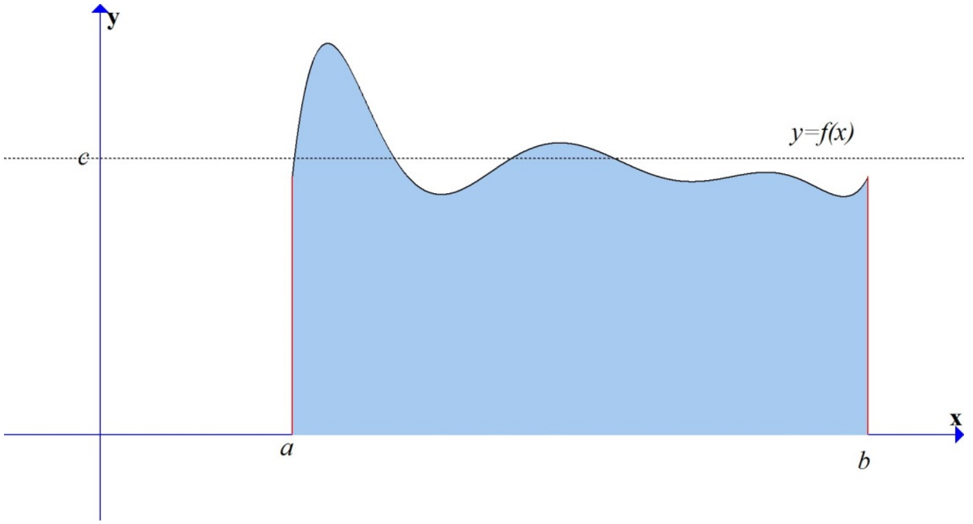


- Average value of $f = \frac{\text{Area under } f \text{ over } [a,b]}{\text{Width of interval}} = \frac{c(b-a)}{b-a} = c$

Average Value

Suppose f is continuous on $[a, b]$, $f(x) \geq 0$ for all x in $[a, b]$ and that A is the area under the curve $y = f(x)$ over that interval. Then the **average value of $\boldsymbol{f}$ over $[\boldsymbol{a}, \boldsymbol{b}]$** is:

$$\text{av}(f) = \frac{A}{b-a}$$



Example

Use a finite sum to estimate the average value of $f(x) = \cos(x)$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by partitioning the interval into four subintervals of equal length and using the lower sum method.

<u>Solution</u>:

First, we need to find the area under $f(x) = \cos(x)$ over $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Example (continued)

$$f(x) = \cos(x)$$

$$a = -\frac{\pi}{2} \text{ and } b = \frac{\pi}{2}$$

$$n = 4$$

$$\Delta x = \frac{\text{width of the interval}}{\text{number of subintervals}} = \frac{b-a}{n}$$

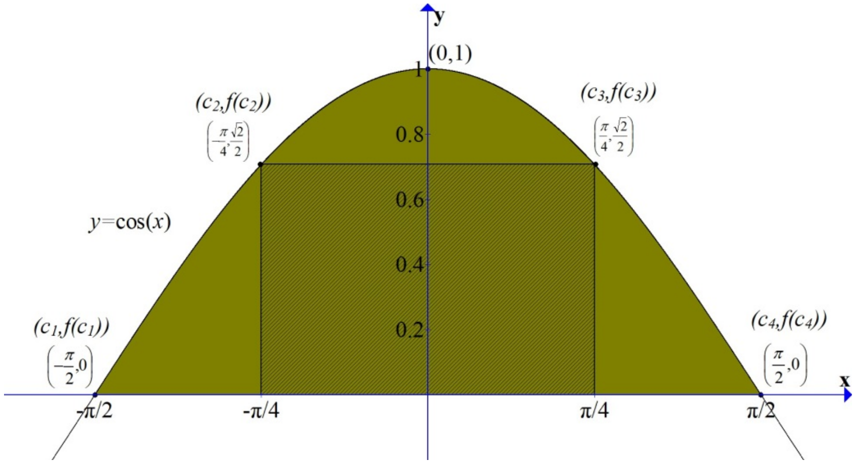
$$\Delta x = \frac{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)}{4} = \frac{\pi}{4}$$

Example (continued)

$$A \approx f(c_1)\cdot\Delta x + f(c_2)\cdot\Delta x + f(c_3)\cdot\Delta x + f(c_4)\cdot\Delta x$$

$$A \approx f\left(-\frac{\pi}{2}\right)\cdot\frac{\pi}{4} + f\left(-\frac{\pi}{4}\right)\cdot\frac{\pi}{4} + f\left(\frac{\pi}{4}\right)\cdot\frac{\pi}{4} + f\left(\frac{\pi}{2}\right)\cdot\frac{\pi}{4}$$

$$\text{A} \approx 0\cdot\frac{\pi}{4} + \frac{\sqrt{2}}{2}\cdot\frac{\pi}{4} + \frac{\sqrt{2}}{2}\cdot\frac{\pi}{4} + 0\cdot\frac{\pi}{4} = \frac{\sqrt{2}\cdot\pi}{4}$$



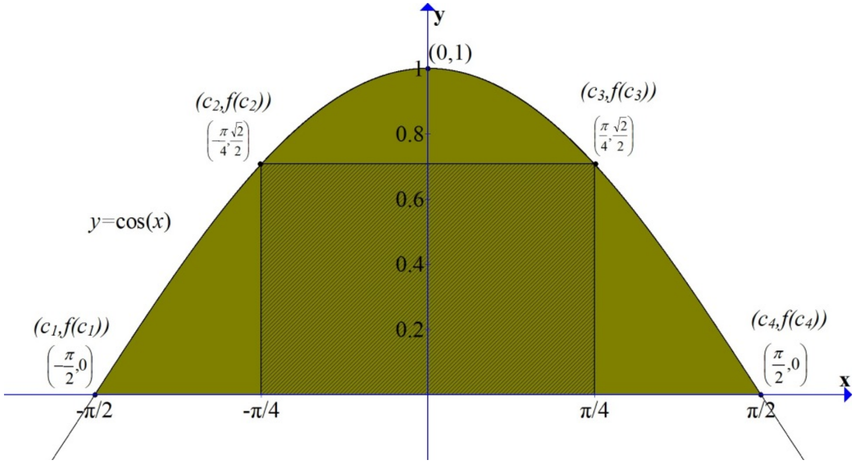
Example (continued)

Now we can find the average value:

$$\text{av}(f) = \frac{A}{b-a} \approx \frac{\frac{\sqrt{2}\cdot\pi}{4}}{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)}$$

$$= \frac{\sqrt{2}}{4}$$

Answer: The average value of $y = \cos(x)$ over the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is approximately $\frac{\sqrt{2}}{4}$.



Girl: How do I look? ;">

Boy: $\frac{\tan c}{\sin c}$

Girl: Huh?

Boy: $\frac{\tan c}{\sin c}$

$$= \frac{\left(\frac{\sin c}{\cos c}\right)}{\sin c}$$

$$= \frac{1}{\cos c}$$

$= \sec c!$ =))