Name ______________________________ Section ______________

Student ID Number ______________ Instructor ______________________

Instructions: No note or calculators are allowed. Answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. **Work must be neat, organized and easily interpreted.**

A page of useful information has been included on the last page of the exam.

Please circle your final answers.

Notes:
$0! = 1$ and if $n > 0$ then $n! = 1 \times 2 \times 3 \times \cdots \times n$

Page #	**Point Value**	**Grade**
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10	10	
Total	100	

1. Evaluate the integrals:
    a. (6 pts) $\int_0^1 \frac{9x^2\,dx}{\sqrt{1-x^3}}$

    b. (6 pts) $\int \sin^{2017}(2x)\cos^3(2x)\,dx$

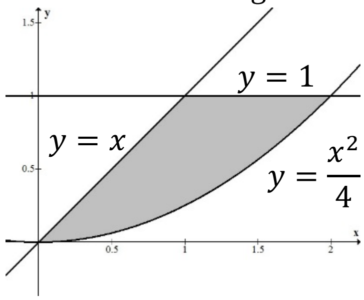
c. (6 pts) $\int x^3 \ln(x)\, dx$

d. (6 pts) $\int \frac{1}{\sqrt{9+x^2}} dx$

e. (6 pts) $\int \frac{x+4}{x^2+5x-6}\,dx$

f. (6 pts) $\int_1^\infty \frac{1}{1+x^2}\,dx$

2. Consider the shaded region:



Set up, but do NOT evaluate, the integral or sum of integrals needed to find:

a. (6 pts) the volume of the solid generated by revolving the shaded region about the **x-axis**.

b. (6 pts) the perimeter of the shaded region.
(Hint: Determine the length of $y = x$ from $(0,0)$ to $(1,1)$; determine the length of $y = 1$ from $(1,1)$ to $(2,1)$; and find the integral that determines the length of $y = \frac{x^2}{4}$ from $(0,0)$ to $(2,1)$. Then add these three lengths together).

3. Determine whether the following series **converge** or **diverge**. Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

   a. (5 pts) $\sum_{k=0}^{\infty} \frac{e^k}{e^k+k}$

   b. (5 pts) $\sum_{n=2}^{\infty} \frac{\sqrt{n}(n+1)}{(n^2+1)(n-1)}$

c. (5 pts) $\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2}$

d. (5 pts) $\sum_{k=1}^{\infty} \frac{(-1)^k}{1+\sqrt{k}}$

4. (10 pts) Find the center, the radius of convergence and the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(3x+1)^{n+1}}{2n+2}$$

Center: ____________________ Radius of convergence: ____________________

Interval of convergence: ____

Don't forget to check the endpoints of the interval of convergence!

5. (6 pts) Find the Taylor series generated by $f(x) = x^4 + x^2 + 1$ at $x = -2$.

6. (6 pts) The Maclaurin series for $\ln(1 + x)$ is

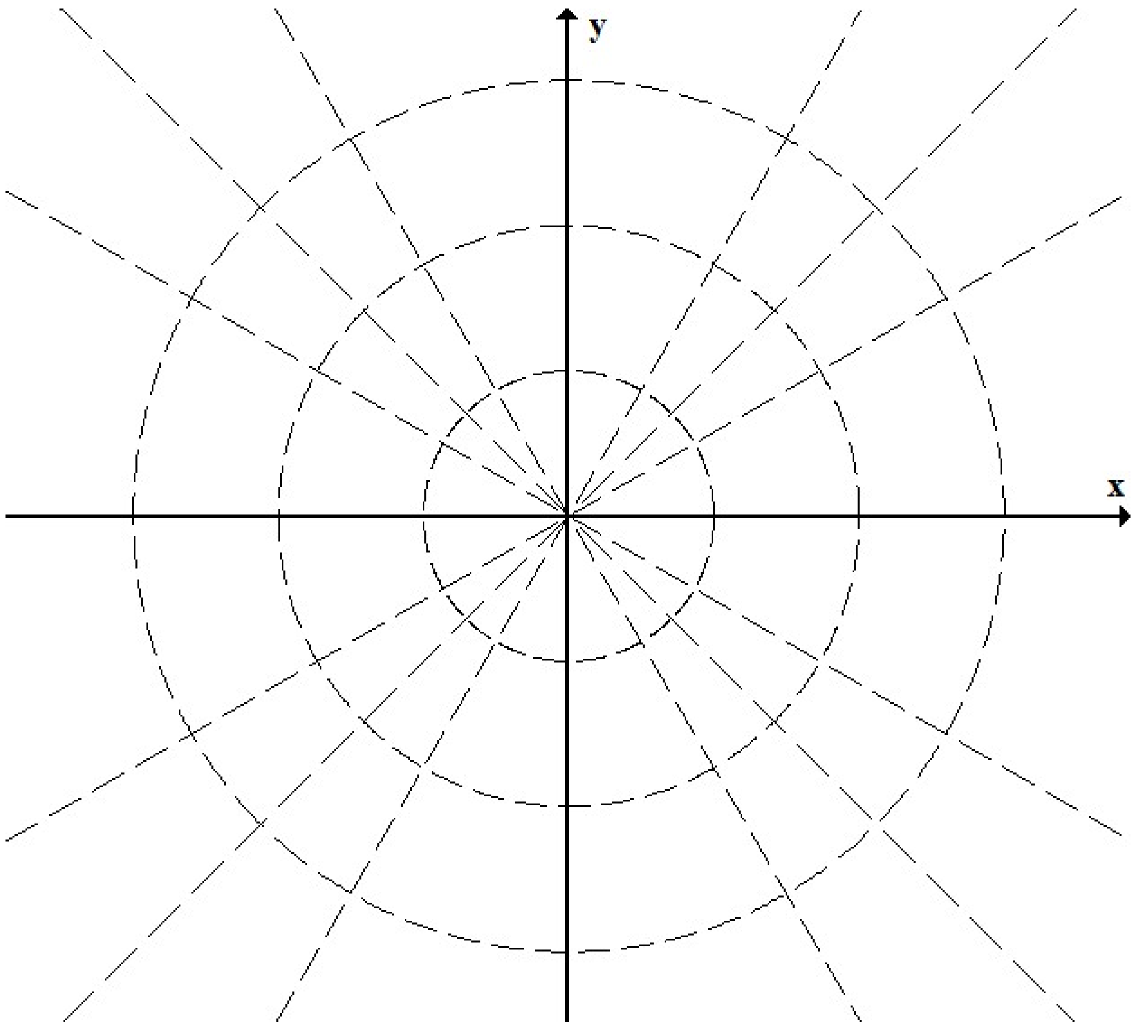
$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}, \qquad -1 < x \leq 1$$

Find the Maclaurin series for $g(x) = \ln(1 + 9x^2)$.

7. (10 pts) The polar curve

$$r = 2 - \cos(2\theta)$$

is symmetric about the x-axis. Plot this polar curve on the grid provided.



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Disk Method: $V = \int_a^b \pi[R(x)]^2\,dx$

Washer Method: $V = \int_a^b \pi([R(x)]^2 - [r(x)]^2)\,dx$

Shell Method: $V = \int_a^b 2\pi r(x)h(x)\,dx$

Surface Area: $S = \int_a^b 2\pi \cdot f(x)\sqrt{1 + \left(f'(x)\right)^2}\,dx$

Arc Length Formula: $L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$

Useful Trigonometric Identities: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; $\sin 2\theta = 2\sin\theta\cos\theta$

The n-th Term Test for Divergence: $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

The Limit Comparison Test: Let $\sum a_n$ and $\sum b_n$ be series with positive terms and suppose $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.

a) If L is finite and $L > 0$, then the series both converge or both diverge.

b) If $L = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

c) If $L = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Ratio Test: Let $\sum a_n$ be a series with nonzero terms and suppose $\rho = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$.

a) If $\rho < 1$, the series converges absolutely.

b) If $\rho > 1$ or $\rho = \infty$, the series diverges.

c) If $\rho = 1$, then the test is inconclusive, use a different test.

Alternating Series Test: An alternating series $\sum_{n=1}^{\infty}(-1)^n u_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1}u_n$ converges if the following three conditions are satisfied:

1) $u_n > 0$ for all $n \geq N$ 2) $\lim_{n\to\infty} u_n = 0$ 3) $u_n \geq u_{n+1}$ for all $n \geq N$ for some N

The **n-th Taylor polynomial** for f about $x = a$ is $p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

The **Taylor series** for f about $x = a$ is $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

Symmetry Tests for Polar Graphs

1. *Symmetry about the x-axis:* If the point (r,θ) lies on the graph, then $(r,-\theta)$ or $(-r,\pi-\theta)$ also lies on the graph.
2. *Symmetry about the y-axis:* If the point (r,θ) lies on the graph, then $(r,\pi-\theta)$ or $(-r,-\theta)$ also lies on the graph.
3. *Symmetry about the origin:* If the point (r,θ) lies on the graph, then $(-r,\theta)$ or $(r,\theta+\pi)$ also lies on the graph.

Area Enclosed by a Polar Curve: $A = \int_\alpha^\beta \frac{1}{2}r^2\,d\theta$

Arc Length of a Polar Curve: $L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$