MATH.1320 Calculus II Spring 2016 Final Exam

Name ____Section __________________

Student ID Number ____________________ Instructor __________________________________

Instructions: No note or calculators are allowed. Answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. **Work must be neat, organized and easily interpreted.**

A page of useful information has been included on the last page of the exam.

Please circle your final answers.

Notes:
$0! = 1$ and if $n > 0$ then $n! = 1 \times 2 \times 3 \times \cdots \times n$

Page #	**Point Value**	**Grade**
3	12	
4	12	
5	10	
6	12	
7	12	
8	12	
9	6	
10	8	
11	11	
12	5	
Total	100	

THIS PAGE IS INTENTIONALLY BLANK

1. Evaluate the integrals:
    a. (6 pts) $\int_0^1 x\sqrt{3x^2+1}\ dx$

    b. (6 pts) $\int \cos^3(x)\ dx$

c. (6 pts) $\int x\cos(7x)\ dx$

d. (6 pts) $\int \frac{30}{x^2-25x+100}\ dx$

2. (5 pts) Suppose that you use the substitution $x = 5\tan\theta$ on an integral $I = \int f(x)\,dx$ and the integral in terms of θ is equal to

$$I = \sin\theta + \sec\theta + C$$

Find an expression for I in terms of x that does not include any inverse trigonometric functions.

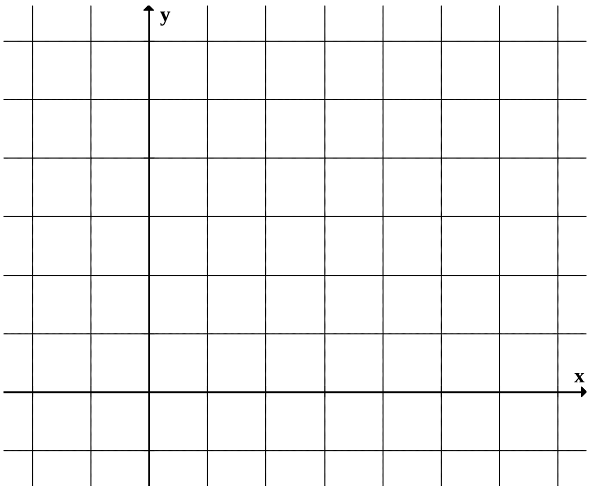
3. (5 pts) Write the following improper integral in terms of the limit of a proper integral. Then determine whether the integral converges or diverges. If the integral converges, evaluate it.

$$\int_0^\infty e^{4x}\ dx$$

4. (6 pts) Sketch and shade in the region **in the first quadrant** bounded by the curves

$$y = \sqrt[3]{x} \text{ and } y = x^3.$$

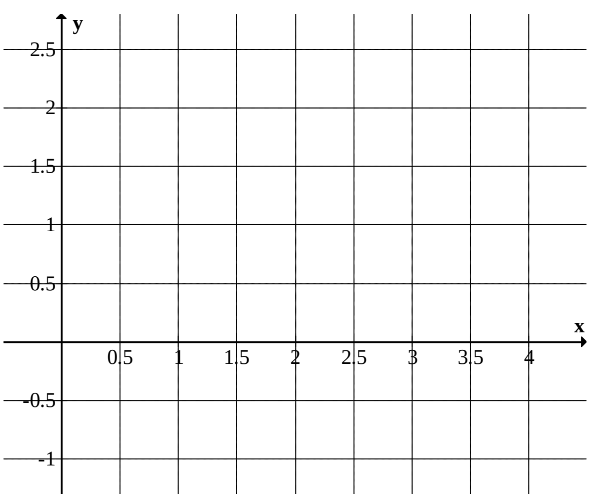
Find the area of this region.



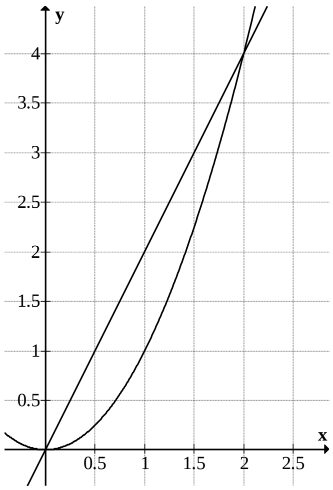
5. (6 pts) Sketch and shade in the region bounded by the x-axis and the curves

$$y = \sqrt{x}, \qquad x = 1 \qquad \text{and} \qquad x = 3.$$

Find the volume generated by revolving this region about the $\underline{x\text{-axis}}$.



6. (6 pts) Let R be the region in the first quadrant bounded above by $y = 2x$ and below by $y = x^2$. Set up, **but do not evaluate**, the sum of integrals that represent the perimeter of R.



7. (6 pts) The value of π can be calculated as follows:

$$\pi = 4\int_0^1 \frac{1}{1+x^2}\,dx$$

Use Simpson's Rule with $n = 4$ to estimate the value of π. Do **not** simplify your answer.

8. Determine whether the following series **converge** or **diverge**. Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

a. (6 pts) $\sum_{k=0}^{\infty} \frac{5^k}{1+6^k}$

b. (6 pts) $\sum_{n=0}^{\infty} \frac{n^5}{n^5+2016}$

9. (6 pts) Find the radius of convergence for the power series

$$A = \sum_{k=1}^{\infty} \frac{(x-2)^k}{k^3 3^k}.$$

10. (8 pts) The power series

$$A = \sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{2n+1}}{2n+1}$$

has interval of convergence $0 \leq x \leq 2$.

a. For which values of x does the power series A converge absolutely? Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

b. For which values of x does the power series A converge conditionally?

11. Maclaurin and Taylor series

a. (6 pts) Find the Taylor series generated by $f(x) = x^4$ centered at $x = -1$.

b. (5 pts) Using the fact that the Maclaurin series for

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot x^{2n+1},$$

find the Maclaurin series for $2 \sin x \cos x$. Give your answer in **summation form**. (Hint: A trigonometric identity is very useful here.)

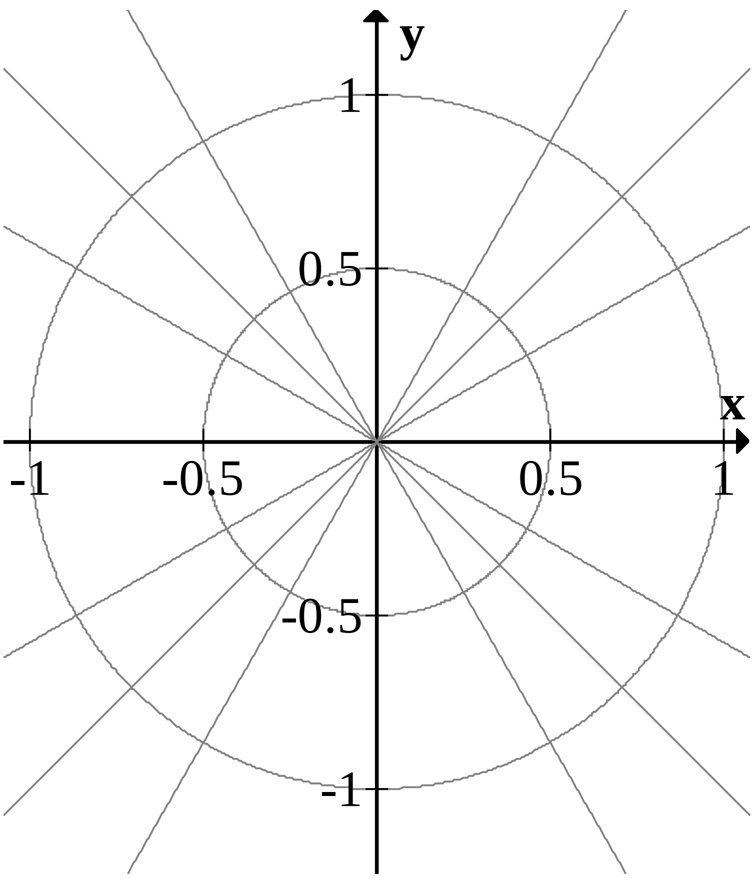
12. (5 pts) On the grid provided, plot the polar curve

$$r = \sin 2\theta.$$

On the grid provided, mark and label the nine points on the curve where

$$\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi.$$

NOTE: $\sqrt{2}/2 \approx 0.7$ $\sqrt{3}/2 \approx 0.9$



THIS PAGE IS INTENTIONALLY BLANK

Disk Method: $V = \int_a^b \pi[R(x)]^2\,dx$

Washer Method: $V = \int_a^b \pi([R(x)]^2 - [r(x)]^2)\,dx$

Shell Method: $V = \int_a^b 2\pi r(x)h(x)\,dx$

Surface Area: $S = \int_a^b 2\pi \cdot f(x)\sqrt{1 + \left(f'(x)\right)^2}\,dx$

Arc Length Formula: $L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$

Useful Trigonometric Identities: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; $\sin 2\theta = 2\sin\theta\cos\theta$

The n-th Term Test for Divergence: $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

The Limit Comparison Test: Let $\sum a_n$ and $\sum b_n$ be series with positive terms and suppose $\rho = \lim_{n\to\infty} \frac{a_n}{b_n}$.

a) If ρ is finite and $\rho > 0$, then the series both converge or both diverge.

b) If $\rho = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

c) If $\rho = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Ratio Test: Let $\sum u_n$ be a series with nonzero terms and suppose $\rho = \lim_{n\to\infty} \frac{|u_{n+1}|}{|u_n|}$.

a) If $\rho < 1$, the series converges absolutely.

b) If $\rho > 1$ or $\rho = \infty$, the series diverges.

c) If $\rho = 1$, the series may converge or diverge.

Alternating Series Test: An alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converges if the following three conditions are satisfied:

1) $a_n > 0$ for all n 2) $\lim_{n\to\infty} a_n = 0$ 3) $a_n \geq a_{n+1}$ for all $n \geq N$ for some N

The **n-th Taylor polynomial** for f about $x = a$ is $p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

The **Taylor series** for f about $x = a$ is $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

Symmetry Tests for Polar Graphs

1. *Symmetry about the x-axis:* If the point (r, θ) lies on the graph, then $(r, -\theta)$ or $(-r, \pi - \theta)$ also lies on the graph.
2. *Symmetry about the y-axis:* If the point (r, θ) lies on the graph, then $(r, \pi - \theta)$ or $(-r, -\theta)$ also lies on the graph.
3. *Symmetry about the origin:* If the point (r, θ) lies on the graph, then $(-r, \theta)$ or $(r, \theta + \pi)$ also lies on the graph.

Area Enclosed by a Polar Curve: $A = \int_\alpha^\beta \frac{1}{2} r^2\,d\theta$

Arc Length of a Polar Curve: $L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$