92.132 Calculus II Spring 2015 Final Exam

Name ______________________________________________Section __________________

Student ID Number ____________________ Instructor __________________________________

**Instructions:** No note or calculators are allowed. Answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. **Work must be neat, organized and easily interpreted.**

**A page of useful information has been included on the last page of the exam.**

**Please circle your final answers.**

Notes:
$0! = 1$ and if $n > 0$ then $n! = 1 \times 2 \times 3 \times \cdots \times n$

| Number | Point Value | Grade |
|---|---|---|
| 1 | 12 | |
| 2 | 12 | |
| 3 | 7 | |
| 4 | 7 | |
| 5 | 10 | |
| 6 | 7 | |
| 7 | 6 | |
| 8 | 6 | |
| 9 | 18 | |
| 10 | 10 | |
| 11 | 5 | |
| Total | 100 | |

1. (12 pts) Evaluate the definite integrals:

   a. $\int_0^{\pi/2} x\sin(4x)\,dx$

   b. $\int_0^{\pi/4} \cos^3 x\,dx$

2. (12 pts) Evaluate the indefinite integrals.

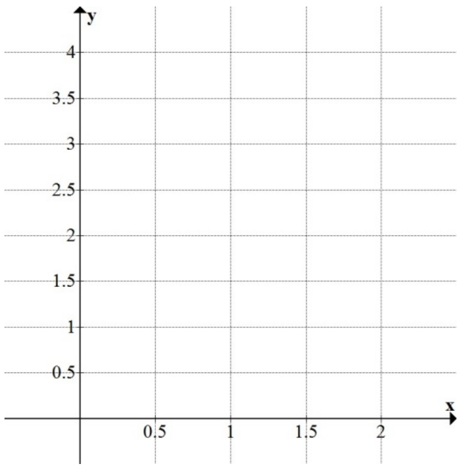
   a. $\int \frac{x^2\,dx}{\sqrt{x^3+1}}$

   b. $\int \frac{10x+3}{2x(2x+1)}\,dx$ (Hint: Use partial fractions.)

3. (7 pts) Sketch the region in the first quadrant enclosed by

$$y = x^2, \qquad y = 2 + x, \qquad x = 0.$$

Set up, but do not evaluate, an integral or sum of integrals that gives the volume generated by revolving this region about the <u>$x$-axis</u>.




4. (7 pts) Find the length of the curve

$$y = \frac{2}{3}(x^2 + 1)^{3/2}, 0 \leq x \leq 1$$

5. (10 pts) Using trigonometric substitution, evaluate

$$\int \frac{1}{x^2\sqrt{16-x^2}}\,dx$$

6. (7 pts) Write the following improper integral in terms of the limit of a proper integral. Then determine whether the integral converges or diverges. If the integral converges, evaluate it.

$$\int_{-1}^{+\infty} \frac{2x}{1+x^2}\,dx$$

7. (6 pts) Estimate the value of

$$\int_0^4 x^2\, dx$$

using Simpson's Rule with 4 partitions.

8. (6 pts) What is the 3rd Taylor polynomial for $f(x) = \ln x$ about $x = 1$?

9. (18 pts) Determine whether the following series converge or diverge. Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

a) $\sum_{k=0}^{\infty} \frac{1}{9^k+1}$

b) $\sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{n!}$

c) $\sum_{n=1}^{\infty} \ln\left(\frac{3n}{n+1}\right)$

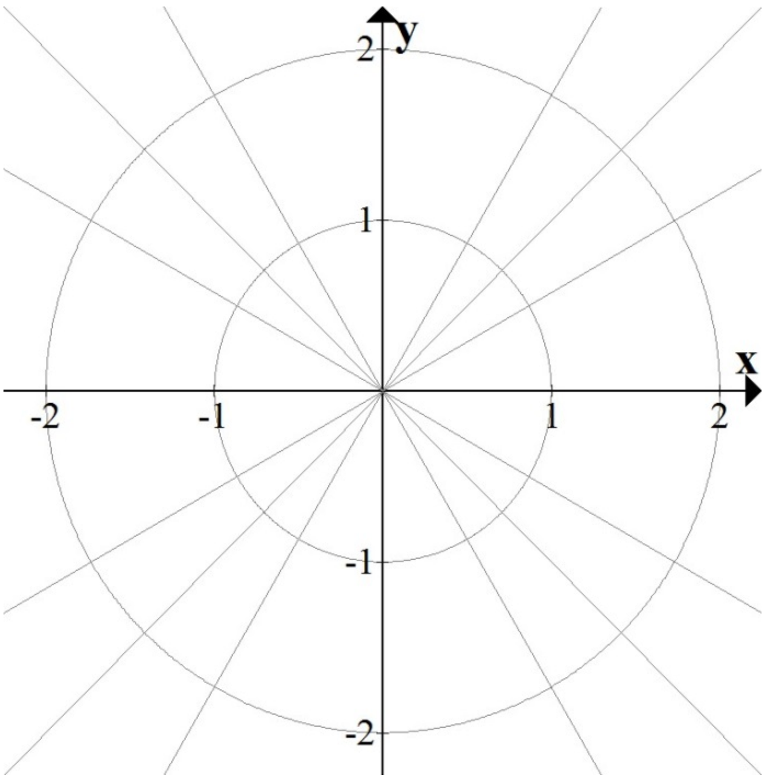
10. (10 pts) Consider the power series $A = \sum_{n=0}^{\infty} (-1)^n \frac{(x-4)^n}{(n+1)^2}$.

a. What is the interval of convergence of $A$? (Note: Remember to check the endpoints of the interval of convergence.)

b. What is the radius of convergence of $A$?

11. (5 pts) On the grid provided, plot the polar curve $r = -2\cos\theta + 2$ for values of $\theta$ ranging from 0 to $\frac{\pi}{2}$. <u>Mark and label</u> the five points on the curve where $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$.

NOTE: $\sqrt{2}/2 \approx 0.7$ $\sqrt{3}/2 \approx 0.9$



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**Disk Method**: $V = \int_a^b \pi[R(x)]^2\, dx$

**Washer Method**: $V = \int_a^b \pi([R(x)]^2 - [r(x)]^2)\, dx$

**Shell Method**: $V = \int_a^b 2\pi rh\, dx$

**Surface Area**: $S = \int_a^b 2\pi \cdot f(x)\sqrt{1 + \left(f'(x)\right)^2}\, dx$

**Arc Length Formula**: $L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\, dx$

**Useful Trigonometric Identities**: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; $\sin 2\theta = 2\sin\theta\cos\theta$

**The $n$-th Term Test for Divergence**: $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

**The Limit Comparison Test**: Let $\sum a_n$ and $\sum b_n$ be series with positive terms and suppose $\rho = \lim_{n\to\infty} \frac{a_n}{b_n}$.

a) If $\rho$ is finite and $\rho > 0$, then the series both converge or both diverge.

b) If $\rho = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

c) If $\rho = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

**Ratio Test**: Let $\sum u_n$ be a series with nonzero terms and suppose $\rho = \lim_{n\to\infty} \frac{|u_{n+1}|}{|u_n|}$.

a) If $\rho < 1$, the series converges absolutely.

b) If $\rho > 1$ or $\rho = \infty$, the series diverges.

c) If $\rho = 1$, the series may converge or diverge.

**Alternating Series Test**: An alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converges if the following three conditions are satisfied:

1) $a_n > 0$ for all $n$  2) $\lim_{n\to\infty} a_n = 0$  3) $a_n \geq a_{n+1}$ for all $n \geq N$ for some $N$

The **$n$-th Taylor polynomial** for $f$ about $x = a$ is $p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

The **Taylor series** for $f$ about $x = a$ is $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

**Symmetry Tests for Polar Graphs**

1. *Symmetry about the x-axis:* If the point $(r, \theta)$ lies on the graph, then $(r, -\theta)$ or $(-r, \pi - \theta)$ also lies on the graph.
2. *Symmetry about the y-axis:* If the point $(r, \theta)$ lies on the graph, then $(r, \pi - \theta)$ or $(-r, -\theta)$ also lies on the graph.
3. *Symmetry about the origin:* If the point $(r, \theta)$ lies on the graph, then $(-r, \theta)$ or $(r, \theta + \pi)$ also lies on the graph.

**Area Enclosed by a Polar Curve**: $A = \int_\alpha^\beta \frac{1}{2} r^2\, d\theta$

**Arc Length of a Polar Curve**: $L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$