Name ____Section __________________

Student ID Number ___________________ Instructor ________________________________

Instructions: No note or calculators are allowed. Answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. **Work must be neat, organized and easily interpreted.**

A page of useful information has been included on the last page of the exam.

Please circle your final answers.

Notes:
$0! = 1$ and if $n > 0$ then $n! = 1 \times 2 \times 3 \times \cdots \times n$

Page #	Point Value	Grade
3	14	
4	7	
5	7	
6	7	
7	7	
8	7	
9	12	
10	12	
11	7	
12	7	
13	7	
14	6	
Total	100	

THIS PAGE IS INTENTIONALLY BLANK

1. Evaluate the integrals:
   a. (7 pts) $\int_0^1 \frac{x^{1/6}\,dx}{\sqrt{12x^{7/6}+4}}$

   b. (7 pts) $\int xe^{2x+3}\,dx$

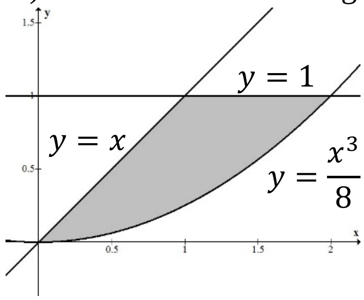
c. (7 pts) $\int \sin^2(7x)\cos^5(7x)\,dx$

d. (7 pts) $\int \frac{1}{x^2\sqrt{x^2-25}}\,dx$

e. (7 pts) $\int \frac{1}{(x-1)(x+2)(x-3)}\,dx$

f. (7 pts) $\int_{-\infty}^{0} \frac{1}{(2x-1)^3} \, dx$

2. (7 pts) Consider the shaded region:



Find the volume of the solid generated by revolving the shaded region about the ***x*-axis**.

3. Determine whether the following series **converge** or **diverge**. Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

   a. (6 pts) $\sum_{k=1}^{\infty} \ln\left(\frac{1}{k}\right)$

   b. (6 pts) $\sum_{n=1}^{\infty} \frac{n(n+3)}{(n+1)(n+2)(n+5)}$

c. (6 pts) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot n\left(\frac{2}{3}\right)^{n}$

d. (6 pts) $\sum_{k=1}^{\infty} (-1)^{k} e^{-k}$

4. (7 pts) Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^{2n-2}}{(2n+1)!}$$

Interval of Convergence: ___________________

5. (7 pts) Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} 2^n(x-3)^n$$

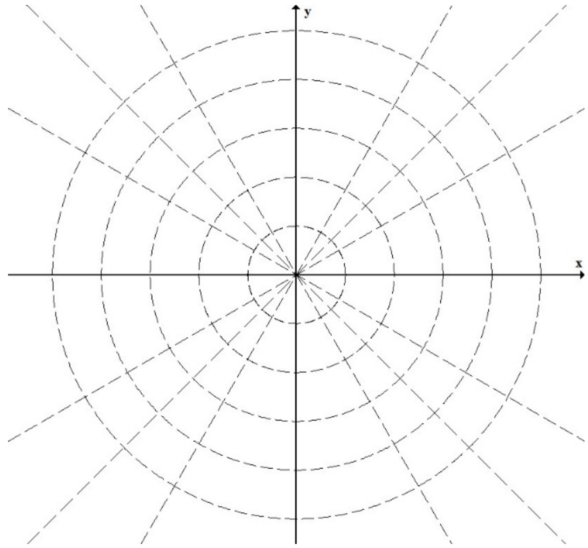
Also, find the sum of the series (as a function of x) on its interval of convergence.

Interval of Convergence: ___________________ Sum of the series: _____________________

6. (7 pts) Find the Taylor **series** generated by $f(x) = e^x$ at $a = 2$.

7. (4 pts) Find the Cartesian coordinates of the following points given in polar coordinates and sketch them on the given coordinate system.

   a. $(r, \theta) = (-3, \pi)$



   b. $(r, \theta) = \left(5, \frac{4\pi}{3}\right)$

8. (2 pts) Find the polar coordinates, $0 \leq \theta < 2\pi$ and $r \geq 0$, of the following points given in Cartesian coordinates.

   a. $(x, y) = \left(\sqrt{2}, \sqrt{2}\right)$

   b. $(x, y) = \left(-4\sqrt{3}, 4\right)$

THIS PAGE IS INTENTIONALLY BLANK

Disk Method: $V = \int_a^b \pi[R(x)]^2\,dx$

Washer Method: $V = \int_a^b \pi([R(x)]^2 - [r(x)]^2)\,dx$

Shell Method: $V = \int_a^b 2\pi r(x)h(x)\,dx$

Surface Area: $S = \int_a^b 2\pi \cdot f(x)\sqrt{1 + \left(f'(x)\right)^2}\,dx$

Arc Length Formula: $L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$

Useful Trigonometric Identities: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; $\sin 2\theta = 2\sin\theta\cos\theta$

The n-th Term Test for Divergence: $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

The Limit Comparison Test: Let $\sum a_n$ and $\sum b_n$ be series with positive terms and suppose $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.

a) If L is finite and $L > 0$, then the series both converge or both diverge.

b) If $L = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

c) If $L = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Ratio Test: Let $\sum a_n$ be a series with nonzero terms and suppose $\rho = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$.

a) If $\rho < 1$, the series converges absolutely.

b) If $\rho > 1$ or $\rho = \infty$, the series diverges.

c) If $\rho = 1$, then the test is inconclusive, use a different test.

Alternating Series Test: An alternating series $\sum_{n=1}^{\infty}(-1)^n u_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} u_n$ converges if the following three conditions are satisfied:

1) $u_n > 0$ for all $n \geq N$ 2) $\lim_{n\to\infty} u_n = 0$ 3) $u_n \geq u_{n+1}$ for all $n \geq N$ for some N

The **n-th Taylor polynomial** for f about $x = a$ is $p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

The **Taylor series** for f about $x = a$ is $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

Symmetry Tests for Polar Graphs

1. *Symmetry about the x-axis:* If the point (r, θ) lies on the graph, then $(r, -\theta)$ or $(-r, \pi - \theta)$ also lies on the graph.
2. *Symmetry about the y-axis:* If the point (r, θ) lies on the graph, then $(r, \pi - \theta)$ or $(-r, -\theta)$ also lies on the graph.
3. *Symmetry about the origin:* If the point (r, θ) lies on the graph, then $(-r, \theta)$ or $(r, \theta + \pi)$ also lies on the graph.

Area Enclosed by a Polar Curve: $A = \int_\alpha^\beta \frac{1}{2} r^2\,d\theta$

Arc Length of a Polar Curve: $L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$