Name ____Section ________________

Student ID Number ___________________ Instructor ________________________________

Instructions: No note or calculators are allowed. Answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. **Work must be neat, organized and easily interpreted.**

A page of useful information has been included on the last page of the exam.

Please circle your final answers.

Notes:
$0! = 1$ and if $n > 0$ then $n! = 1 \times 2 \times 3 \times \cdots \times n$

Page #	**Point Value**	**Grade**
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Total	100	

1. Evaluate the integrals:
    a. (6 pts) $\int_0^{\pi} \frac{\sin x}{2-\cos x}\, dx$

    b. (6 pts) $\int x^2 \sin x \;\, dx$

c. (6 pts) $\int \frac{5x-3}{x^2-2x-3}\, dx$

d. (6 pts) $\int_0^\infty \frac{1}{1+x^2}\, dx$

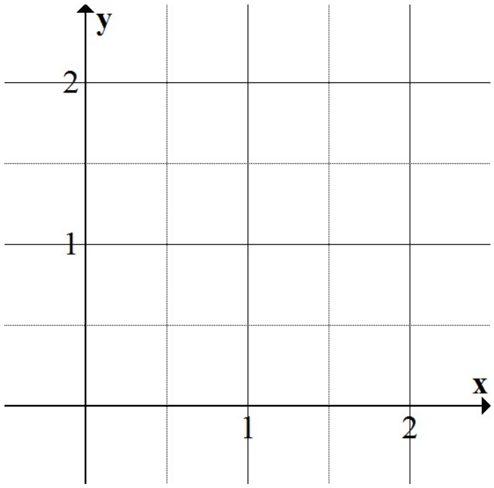
2. (10 pts) Use trigonometric substitution to evaluate

$$\int \frac{x^3}{\sqrt{1-x^2}}\, dx.$$

3. (6 pts) Sketch and shade in the region bounded by the curves

$$y = x, \qquad y = \frac{1}{x^2} \quad \text{and} \quad x = 2.$$

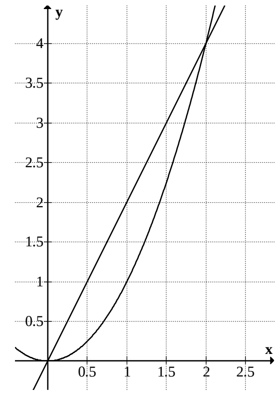
Find the **area** of this region.



4. (6 pts) Find the **area of the surface** generated by revolving the curve below about the x-axis.

$$y = \sqrt{2x - x^2}, \qquad \frac{1}{2} \le x \le \frac{3}{2}$$

5. Let R be the region in the first quadrant bounded above by $y = 2x$ and below by $y = x^2$.



a. (6 pts) Find the volume of the solid generated when R is revolved about the ***x*-axis**.

b. (6 pts) Find the volume of the solid generated when R is revolved about the ***y*-axis**.

6. Determine whether the following series **converge** or **diverge**. Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

   a. (6 pts) $\sum_{k=1}^{\infty} \frac{k+1}{k^2\sqrt{k}}$

   b. (6 pts) $\sum_{n=0}^{\infty} \frac{(-1)^n \cdot n^{23}}{n^{23}+1}$

7. (10 pts) Find the radius and interval of convergence for the power series

$$\sum_{k=0}^{\infty} \frac{(2016)^k x^k}{k!}.$$

a. Radius of convergence: ______________________________

b. Interval of convergence: ______________________________

8. (6 pts) Find the Taylor series generated by $f(x) = x^4$ centered at $x = 2$.

9. (6 pts) The Maclaurin series for $\tan^{-1} x$ is

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \cdot x^{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots, \qquad |x| \leq 1$$

Find the Maclaurin series for $g(x) = x \tan^{-1} x$. Give your answer in **summation form**.

10. (2 pts each) Find the Cartesian coordinates of the following points given in polar coordinates.

a. $(r, \theta) = (-1, \pi)$

b. $(r, \theta) = \left(5, \frac{2\pi}{3}\right)$

11. (2 pts each) Find the polar coordinates, $0 \leq \theta < 2\pi$ and $r \geq 0$, of the following points given in Cartesian coordinates.

a. $(x, y) = \left(4\sqrt{3}, 4\right)$

b. $(x, y) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

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Disk Method: $V = \int_a^b \pi[R(x)]^2\,dx$

Washer Method: $V = \int_a^b \pi([R(x)]^2 - [r(x)]^2)\,dx$

Shell Method: $V = \int_a^b 2\pi r(x)h(x)\,dx$

Surface Area: $S = \int_a^b 2\pi \cdot f(x)\sqrt{1 + \left(f'(x)\right)^2}\,dx$

Arc Length Formula: $L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$

Useful Trigonometric Identities: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; $\sin 2\theta = 2\sin\theta\cos\theta$

The n-th Term Test for Divergence: $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

The Limit Comparison Test: Let $\sum a_n$ and $\sum b_n$ be series with positive terms and suppose $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.

a) If L is finite and $L > 0$, then the series both converge or both diverge.

b) If $L = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

c) If $L = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Ratio Test: Let $\sum a_n$ be a series with nonzero terms and suppose $\rho = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$.

a) If $\rho < 1$, the series converges absolutely.

b) If $\rho > 1$ or $\rho = \infty$, the series diverges.

c) If $\rho = 1$, then the test is inconclusive, use a different test.

Alternating Series Test: An alternating series $\sum_{n=1}^{\infty}(-1)^n u_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} u_n$ converges if the following three conditions are satisfied:

1) $u_n > 0$ for all $n \geq N$ 2) $\lim_{n\to\infty} u_n = 0$ 3) $u_n \geq u_{n+1}$ for all $n \geq N$ for some N

The ***n*-th Taylor polynomial** for f about $x = a$ is $p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

The **Taylor series** for f about $x = a$ is $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

Symmetry Tests for Polar Graphs

1. *Symmetry about the x-axis:* If the point (r,θ) lies on the graph, then $(r,-\theta)$ or $(-r,\pi-\theta)$ also lies on the graph.
2. *Symmetry about the y-axis:* If the point (r,θ) lies on the graph, then $(r,\pi-\theta)$ or $(-r,-\theta)$ also lies on the graph.
3. *Symmetry about the origin:* If the point (r,θ) lies on the graph, then $(-r,\theta)$ or $(r,\theta+\pi)$ also lies on the graph.

Area Enclosed by a Polar Curve: $A = \int_\alpha^\beta \frac{1}{2}r^2\,d\theta$

Arc Length of a Polar Curve: $L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$