Name ____ Section __________________

Student ID Number ____________________ Instructor ___________________________________

Instructions: No note or calculators are allowed. Answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. **Work must be neat, organized and easily interpreted.**

A page of useful information has been included on the last page of the exam.

Please circle your final answers.

Notes:
$0! = 1$ and if $n > 0$ then $n! = 1 \times 2 \times 3 \times \cdots \times n$

Number	Point Value	Grade
1a-b	10	
1c-d	10	
1e	7	
1f	7	
2-3	10	
4-5	11	
6a-b	10	
6c,7	10	
8	10	
9	10	
10	5	
Total	100	

1. Evaluate the integrals:
   a. (5 pts) $\int x^2\sqrt{x-2}\ dx$ (Hint: Use u-Substitution.)

   b. (5 pts) $\int x^2 \ln 5x\ dx$

c. (5 pts) $\int_0^{\pi/8} \tan^2 2x \sec^2 2x \ dx$

d. (5 pts) $\int \cos^3(x) \sin^2(x) \ dx$

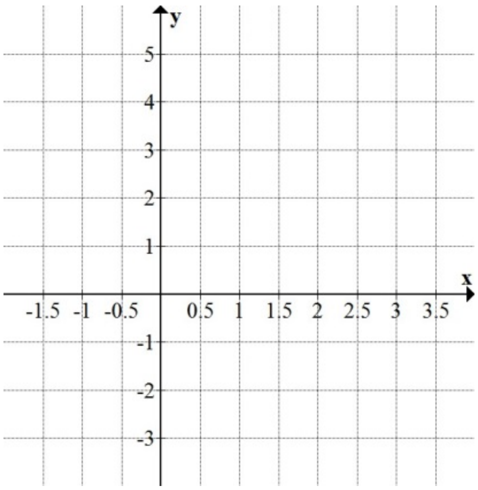
e. (7 pts) $\int \frac{\sqrt{x^2-9}}{x}\,dx$ (Hint: Use Trigonometric Substitution.)

f. (7 pts) $\int \frac{x+3}{(x-2)(x+1)}\,dx$ (Hint: Use Partial Fractions.)

2. (5 pts) Sketch and shade in the region bounded by the curves

$$y = 2x - x^2 \text{ and } y = -3.$$

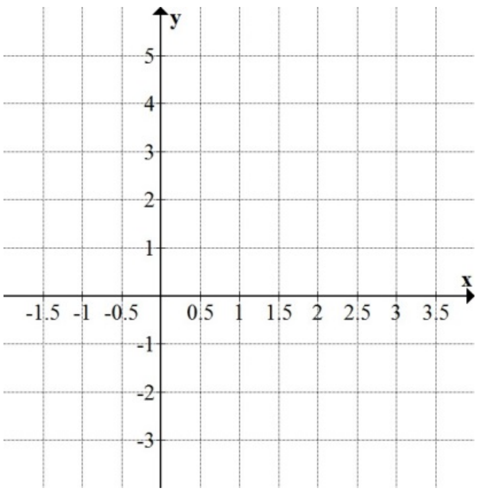
Set up, but do not evaluate, an integral to find the area. Do NOT solve the integral.



3. (5 pts) Sketch and shade in the region bounded by the curves

$$y = x^2 + 1 \text{ and } y = x + 3.$$

Set up, but do not evaluate, an integral or sum of integrals that gives the volume generated by revolving this region about the x-axis. Do NOT solve the integral.



4. (6 pts) Find the area of the surface generated by rotating about the x-axis the arc of the curve $y = \sqrt{x}$ between (0,0) and $\left(2, \sqrt{2}\right)$.

5. (5 pts) Write the following improper integral in terms of the limit of a proper integral. Then determine whether the integral converges or diverges. If the integral converges, evaluate it.

$$\int_0^\infty \frac{1}{2x+1}\, dx$$

6. Determine whether the following series **converge** or **diverge**. Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

   a. (5 pts) $\sum_{n=1}^{\infty} \ln\left(\frac{1}{n}\right)$

   b. (5 pts) $\sum_{k=2}^{\infty} \frac{\sqrt{k}}{k^3-1}$

c. (5 pts) $\sum_{n=1}^{\infty} \frac{(n+3)!}{n!3^n}$

7. (5 pts) The alternating series

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{10^n}$$

satisfies all of the conditions of the Alternating Series Test and therefore converges. Does the series converge absolutely or conditionally? Give a reason for your answer.

8. Power Series
   a. (5 pts) Find the radius of convergence for the series

$$\sum_{k=1}^{\infty} \frac{x^k e^k}{k+1}$$

   b. (5 pts) Find the series' interval of convergence and, within this interval, the sum of the series as a function of x.

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{4^{n+1}}$$

9. Maclaurin and Taylor series
   a. (5 pts) Find the Taylor series generated by $f(x) = x^3$ at $a = 1$.

   b. (5 pts) Using the fact that

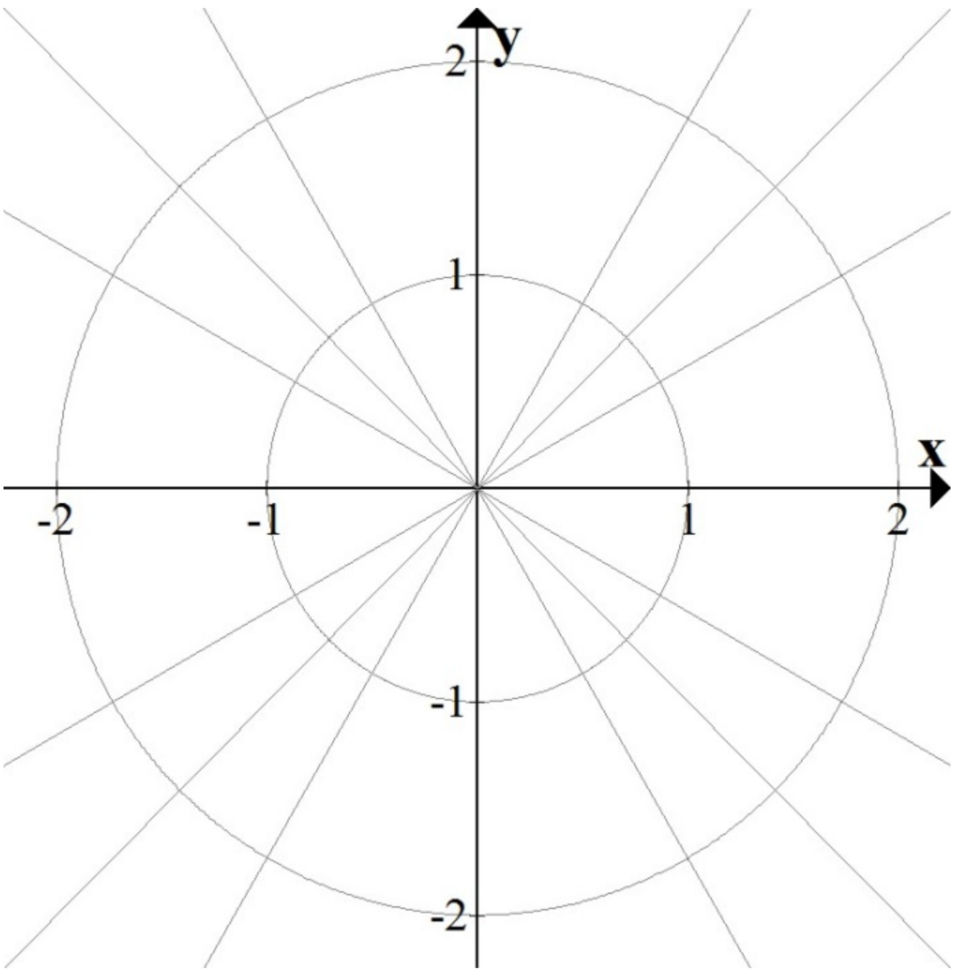
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \qquad |x| < \infty,$$

   find the Maclaurin series for the function

$$f(x) = x^2 e^{2x}$$

10. (5 pts) On the grid provided, plot the polar curve $r = \cos\theta + \sin\theta$. Mark and label the nine points on the curve where $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi$.

NOTE: $\sqrt{2}/2 \approx 0.7$ $\sqrt{3}/2 \approx 0.9$



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Disk Method: $V = \int_a^b \pi[R(x)]^2\,dx$

Washer Method: $V = \int_a^b \pi([R(x)]^2 - [r(x)]^2)\,dx$

Shell Method: $V = \int_a^b 2\pi r(x)h(x)\,dx$

Surface Area: $S = \int_a^b 2\pi \cdot f(x)\sqrt{1 + \left(f'(x)\right)^2}\,dx$

Arc Length Formula: $L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$

Useful Trigonometric Identities: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; $\sin 2\theta = 2\sin\theta\cos\theta$

The n-th Term Test for Divergence: $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

The Limit Comparison Test: Let $\sum a_n$ and $\sum b_n$ be series with positive terms and suppose $\rho = \lim_{n\to\infty} \frac{a_n}{b_n}$.

a) If ρ is finite and $\rho > 0$, then the series both converge or both diverge.

b) If $\rho = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

c) If $\rho = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Ratio Test: Let $\sum u_n$ be a series with nonzero terms and suppose $\rho = \lim_{n\to\infty} \frac{|u_{n+1}|}{|u_n|}$.

a) If $\rho < 1$, the series converges absolutely.

b) If $\rho > 1$ or $\rho = \infty$, the series diverges.

c) If $\rho = 1$, the series may converge or diverge.

Alternating Series Test: An alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converges if the following three conditions are satisfied:

1) $a_n > 0$ for all n 2) $\lim_{n\to\infty} a_n = 0$ 3) $a_n \geq a_{n+1}$ for all $n \geq N$ for some N

The **n-th Taylor polynomial** for f about $x = a$ is $p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

The **Taylor series** for f about $x = a$ is $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

Symmetry Tests for Polar Graphs

1. *Symmetry about the x-axis:* If the point (r,θ) lies on the graph, then $(r,-\theta)$ or $(-r,\pi-\theta)$ also lies on the graph.
2. *Symmetry about the y-axis:* If the point (r,θ) lies on the graph, then $(r,\pi-\theta)$ or $(-r,-\theta)$ also lies on the graph.
3. *Symmetry about the origin:* If the point (r,θ) lies on the graph, then $(-r,\theta)$ or $(r,\theta+\pi)$ also lies on the graph.

Area Enclosed by a Polar Curve: $A = \int_\alpha^\beta \frac{1}{2} r^2\,d\theta$

Arc Length of a Polar Curve: $L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$