EXERCISES

1. **Arithmetic operations:** Compute the following quantities:
   - $\frac{2^5}{2^5-1}$ and compare with $(1-\frac{1}{2^5})^{-1}$.
   - $3\frac{\sqrt{5}-1}{(\sqrt{5}+1)^2}-1$. The square root $\sqrt{x}$ can be calculated with the command `sqrt(x)` or `x^0.5`.
   - Area $=\pi r^2$ with $r=\pi^{\frac{1}{3}}-1$. (π is `pi` in MATLAB.)

2. **Exponential and logarithms:** The mathematical quantities e^x, $\ln x$, and $\log x$ are calculated with `exp(x)`, `log(x)`, and `log10(x)`, respectively. Calculate the following quantities:
   - e^3, $\ln(e^3)$, $\log_{10}(e^3)$, and $\log_{10}(10^5)$.
   - $e^{\pi\sqrt{163}}$.
   - Solve $3^x=17$ for x and check the result. (The solution is $x=\frac{\ln 17}{\ln 3}$. You can verify the result by direct substitution.)

3. **Trigonometry:** The basic MATLAB trig functions are `sin`, `cos`, `tan`, `cot`, `sec`, and `csc`. The inverses, e.g., arcsin, arctan, etc., are calculated with `asin`, `atan`, etc. The same is true for hyperbolic functions. The inverse function `atan2` takes 2 arguments, `y` and `x`, and gives the four-quadrant inverse tangent. The argument of these functions must be in radians.
   Calculate the following quantities:
   - $\sin\frac{\pi}{6}$, $\cos\pi$, and $\tan\frac{\pi}{2}$.
   - $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{6}$. (Typing `sin^2(x)` for $\sin^2 x$ will produce an error).
   - $y=\cosh^2 x-\sinh^2 x$, with $x=32\pi$.

4. **Complex numbers:** MATLAB recognizes the letters `i` and `j` as the imaginary number $\sqrt{-1}$. A complex number $2+5i$ may be input as `2+5i` or `2+5*i` in MATLAB . The former case is always interpreted as a complex number whereas the latter case is taken as complex only if `i` has not been assigned any local value. The same is true for `j`. This kind of context dependence, for better or worse, pervades MATLA[B.]
   Compute the following quantities.
   - $\frac{1+3i}{1-3i}$. Can you check the result by hand calculation?
   - $e^{i\frac{\pi}{4}}$. Check the Euler's Formula $e^{ix}=\cos x+i\sin x$ by co[mputing] the right hand side too, i. e., compute $\cos(\pi/4)+i\sin(\pi/4)$
   - Execute the commands `exp(pi/2*i)` and `exp(pi/2i)`. Can [you] explain the difference between the two results?

Answers to Exercises

1.

Command	Result
`2^5/(2^5-1)`	1.0323
`3*(sqrt(5)-1)/(sqrt(5)+1)^2 - 1`	-0.6459
`area=pi*(pi^(1/3)-1)^2`	0.6781

2.

Command	Result
`exp(3)`	20.0855
`log(exp(3))`	3.0000
`log10(exp(3))`	1.3029
`log10(10^5)`	5.0000
`exp(pi*sqrt(163))`	2.6254e+01
`x=log(17)/log(3)`	2.5789

3.

Command	Result
`sin(pi/6)`	0.5000
`cos(pi)`	-1.0000
`tan(pi/2)`	1.6332e+0
`(sin(pi/6))^2+(cos(pi/6))^2`	1
`x=32*pi; y=(cosh(x))^2-(sinh(x))^2`	0

4.

Command	Result
`(1+3i)/(1-3i)`	-0.8000 +
`exp(i*pi/4)`	0.7071 +
`exp(pi/2*i)`	0.0000 +
`exp(pi/2i)`	0.0000 -

Note that

`exp(pi/2*i)`$= e^{\frac{\pi}{2}i} = \cos(\frac{\pi}{2}) + i\,\sin(\frac{\pi}{2}) = i$

`exp(pi/2i)`$= e^{\frac{\pi}{2i}} = e^{-\frac{\pi}{2}i} = \cos(\frac{\pi}{2}) - i\,\sin(\frac{\pi}{2}) = -i$

EXERCISES

1. **Equation of a straight line:** The equation of a straight line is $y = mx + c$ where m and c are constants. Compute the y-coordinates of a line with slope $m = 0.5$ and the intercept $c = -2$ at the following x-coordinates:

$$x = 0, \quad 1.5, \quad 3, \quad 4, \quad 5, \quad 7, \quad 9, \text{ and } 10.$$

   [Note: Your command should not involve any array operators since your calculation involves multiplication of a vector with a scalar m and then addition of another scalar c.]

2. **Multiply, divide, and exponentiate vectors:** Create a vector t with 10 elements: 1, 2, 3, ..., 10. Now compute the following quantities:

   - $x = t \sin(t)$.
   - $y = \frac{t-1}{t+1}$.
   - $z = \frac{\sin(t^2)}{t^2}$.

3. **Points on a circle:**All points with coordinates $x = r\cos\theta$ and $y = r\sin\theta$, where r is a constant, lie on a circle with radius r, i.e., they satisfy the equation $x^2 + y^2 = r^2$. Create a column vector for θ with the values 0, $\pi/4$, $\pi/2$, $3\pi/4$, π, and $5\pi/4$.

   Take $r = 2$ and compute the column vectors x and y. Now check that x and y indeed satisfy the equation of circle, by computing the radius $r = \sqrt{(x^2 + y^2)}$. [To calculate r you will need the array operator `.^` for squaring x and y. Of course, you could compute x^2 by `x.*x` also.]

4. **The geometric series:** This is funky! You know how to compute x^n element-by-element for a vector x and a scalar exponent n. How about computing n^x, and what does it mean? The result is again a vector with elements n^{x_1}, n^{x_2}, n^{x_3} etc.

   The sum of a geometric series $1 + r + r^2 + r^3 + \ldots + r^n$ approaches the limit $\frac{1}{1-r}$ for $r < 1$ as $n \to \infty$. Create a vector n of 11 elements from 0 to 10. Take $r = 0.5$ and create another vector $x = [r^0 \quad r^1 \quad r^2 \quad \ldots \quad r^n]$ with the command `x = r.^n` . Now take the sum of this vector with the command `s = sum(x)` (`s` is the sum of the actual series). Calculate the limit $\frac{1}{1-r}$ and compare the computed sum `s`. Repeat the procedure taking n from 0 to 50 and then from 0 to 100.

5. **Matrices and vectors:** Go to Fig. 3.1 on page 51 and reproduce the results. Now create a vector and a matrix with the following commands: `v = 0:0.2:12;` and `M = [sin(v); cos(v)];` (see Section 3.1.4 on page 55 for use of ':' in creating vectors). Find the sizes of `v` and `M` using the `size` command. Extract the first 10 elements of each row of the matrix, and display them as column vectors.

Answers to Exercises

Commands to solve each problem are given below.

1.
```
x=[0 1.5 3 4 5 7 9 10];
y = 0.5*x-2
```
Ans. $y = [-2.0000\ \ -1.2500\ \ -0.5000\ \ 0\ \ 0.5000\ \ 1.5000\ \ 2.5000\ \ 3.0$

2.
```
t=1:10;
x = t.*sin(t)
y = (t-1)./(t+1)
z = sin(t.^2)./(t.^2)
```

3.
```
theta = [0;pi/4;pi/2;3*pi/4;pi;5*pi/4]
r = 2;
x=r*cos(theta); y=r*sin(theta);
x.^2 + y.^2
```

4.
```
n = 0:10;
r = 0.5; x = r.^n;
s1 = sum(x)
n=0:50; x=r.^n; s2=sum(x)
n=0:100; x=r.^n; s3=sum(x)
```
[*Ans.* $s1 = 1.9990$, $s2 = 2.0000$, and $s3 = 2$]

5.
```
v=0:0.2:12;
M=[sin(v); cos(v)];
size(v)
size(M)
M(:,1:10)'
```
[*Ans.* v is 1×61 and M is 2×61.
The last command `M(:,1:10)'` picks out the first 10 elements each row of M and transposes to give a 10×2 matrix.]