3.2 Cont'd

(c) Using the two ray model, we can see that at large distances, the received power falls off with distance raised to the fourth power or at a rate of 40dB/decade, and the received power and path loss are independent of frequency.

$\boxed{3.3}$ $\Delta = d'' - d'$

$$= \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

$$= d\left[1 + \left(\frac{h_t + h_r}{d}\right)^2\right]^{\frac{1}{2}} - d\left[1 + \left(\frac{h_t - h_r}{d}\right)^2\right]^{\frac{1}{2}}$$

For $d \gg h_t + h_r$, $\left(\frac{h_t + h_r}{d}\right)^2 \ll 1$, $\left(\frac{h_t - h_r}{d}\right)^2 \ll 1$.

Using Taylor series approximation, we have

$$\Delta \approx d\left[1 + \frac{1}{2}\left(\frac{h_t + h_r}{d}\right)^2\right] - d\left[1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2\right]$$

$$= d \cdot \frac{1}{2}\left[\left(\frac{h_t + h_r}{d}\right)^2 - \left(\frac{h_t - h_r}{d}\right)^2\right]$$

$$= d \cdot \frac{1}{2} \cdot \frac{4 h_t \cdot h_r}{d^2}$$

$$= \frac{2 h_t \cdot h_r}{d}$$

$\boxed{3.4}$ When $d \gg h_t + h_r$, we have $\theta_\Delta = \frac{2\pi}{\lambda} \cdot \frac{2 h_t h_r}{d}$

$$\Rightarrow d = \frac{4\pi}{\lambda} \cdot \frac{h_t h_r}{\theta_\Delta}$$

$$\tan\theta_i = \frac{h_t + h_r}{d} < \tan 5^\circ \Rightarrow \frac{h_t + h_r}{\frac{4\pi}{\lambda} \cdot \frac{h_t h_r}{\theta_\Delta}} < \tan 5^\circ$$

$$\Rightarrow \frac{1 + \frac{h_r}{h_t}}{\frac{4\pi}{\lambda} \cdot \frac{h_r}{\theta_\Delta}} < \tan 5^\circ \Rightarrow h_t > \frac{h_r}{\frac{4\pi \cdot h_r \cdot \tan 5^\circ}{\lambda \cdot \theta_\Delta} - 1}$$

For $h_r = 2m$, $\theta_\Delta = 6.261$, $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{9 \times 10^8} = 0.333m$.

$$\Rightarrow h_t > \frac{2}{\frac{4\pi \times 2 \times \tan 5^\circ}{\frac{1}{3} \times 6.261} - 1} \Rightarrow \underline{\underline{h_{t\,min} = 37.7m}}$$

3.4 Cont'd

$$\Rightarrow d_{min} = \frac{4\pi}{\lambda} \cdot \frac{h_{t\,min} \cdot h_r}{\theta_\Delta} = \frac{4\pi}{0.333} \times \frac{37.7 \times 2}{6.261} \doteq \underline{\underline{453.77\,(m)}}$$

$\boxed{3.5}$ At the location of the signal nulls at the receiver,

$$\theta_\Delta = \frac{2\pi}{\lambda} \cdot \frac{2 h_t h_r}{d} = 2i\pi, \quad i = 1, 2, \cdots$$

$\Rightarrow \underline{\underline{d_{nulls} = \frac{2 h_t \cdot h_r}{i\lambda}}}$, where i is a positive integer such that $d_{nulls} > d_0$.

$\boxed{3.6}$ Approximate : $\underline{\underline{P_r = P_t \cdot G_t \cdot G_r \cdot \frac{h_t^2 \cdot h_r^2}{d^4}}}$

Exact : $|E_{TOT}(d)| = \frac{E_0 \cdot d_0}{d} \cdot \sqrt{2 - 2\cos\theta_\Delta}$

$$\Rightarrow |E_{TOT}(d)|^2 = \frac{E_0^2 \cdot d_0^2}{d^2} \cdot (2 - 2\cos\theta_\Delta)$$

$$P_r(d) = \frac{|E_{TOT}(d)|^2}{120\pi} \cdot A_e = \frac{E_0^2 \cdot d_0^2}{d^2} \cdot (2 - 2\cos\theta_\Delta) \cdot \frac{A_e}{120\pi}$$

$$E_0^2 = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 d_0^2} \cdot \frac{120\pi}{A_e}$$

$$\Rightarrow P_r(d) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d_0^2} \cdot \frac{120\pi}{A_e} \cdot \frac{d_0^2}{d^2} \cdot (2 - 2\cos\theta_\Delta) \cdot \frac{A_e}{120\pi}$$

$$= \underline{\underline{\frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2} \cdot 4 \cdot \sin^2\frac{\theta_\Delta}{2}}}$$

where $\theta_\Delta = \frac{2\pi}{\lambda} \cdot \frac{2 h_t \cdot h_r}{d}$.

See problem 3-14 for the plot.

$\boxed{3.7}$ See problem 3-14 for the plot

$$\Gamma = 1 \Rightarrow \underline{\underline{P_r(d) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2} \cdot 4 \cdot \cos^2\frac{\theta_\Delta}{2}}}$$

$\boxed{3.8}$ We need to find a d_f such that $\Delta = d'' - d' = \frac{\lambda}{2}$.

$$\Delta = d'' - d' = \sqrt{(h_t + h_r)^2 + d_f^2} - \sqrt{(h_t - h_r)^2 + d_f^2}$$

3.8 Cont'd

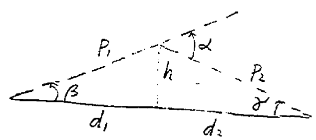
$$\Rightarrow \sqrt{(h_t+h_r)^2+d_f^2} - \sqrt{(h_t-h_r)^2+d_f^2} = \frac{\lambda}{2}$$

$$\Rightarrow (h_t+h_r)^2+d_f^2 = (h_t-h_r)^2+d_f^2+\frac{\lambda^2}{4}+\lambda\cdot\sqrt{(h_t-h_r)^2+d_f^2}$$

$$\Rightarrow d_f = \underline{\underline{\sqrt{\frac{16h_t^2\cdot h_r^2}{\lambda^2}-(h_t^2+h_r^2)+\frac{\lambda^2}{16}}}}$$

$\boxed{3.9}$ (a) $P_1 = \sqrt{d_1^2+h^2} = d_1\sqrt{1+(\frac{h}{d_1})^2}$

$P_2 = \sqrt{d_2^2+h^2} = d_2\sqrt{1+(\frac{h}{d_2})^2}$



Since $d_1, d_2 \gg h \gg \lambda$, $\frac{h}{d_1}, \frac{h}{d_2} \ll 1$, using Taylor series approximation, we have

$$P_1 \doteq d_1\left[1+\frac{1}{2}\left(\frac{h}{d_1}\right)^2\right] = d_1+\frac{1}{2}\frac{h^2}{d_1}$$

$$P_2 \doteq d_2\cdot\left[1+\frac{1}{2}\left(\frac{h}{d_2}\right)^2\right] = d_2+\frac{1}{2}\frac{h^2}{d_2}$$

$$\Rightarrow \Delta = P_1+P_2-(d_1+d_2)$$

$$\doteq \left(d_1+\frac{1}{2}\frac{h^2}{d_1}\right)+\left(d_2+\frac{1}{2}\frac{h^2}{d_2}\right)-(d_1+d_2)$$

$$= \frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 d_2}\right)$$

and $\phi = \frac{2\pi\Delta}{\lambda} \doteq \frac{2\pi}{\lambda}\left[\frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 d_2}\right)\right]$.

(b) From the definition of ν, $\frac{\nu^2\pi}{2} = \phi$

$$\Rightarrow \nu = \sqrt{\phi\cdot\frac{2}{\pi}}$$

$$\Rightarrow \nu = \sqrt{\frac{2\pi}{\lambda}\left[\frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 d_2}\right)\right]\cdot\frac{2}{\pi}} = h\cdot\sqrt{\frac{2}{\lambda}\cdot\frac{d_1+d_2}{d_1 d_2}}$$

Since $\tan\beta = \frac{h}{d_1} \ll 1$, $\tan\gamma = \frac{h}{d_2} \ll 1$, we have

$\beta \approx \tan\beta = \frac{h}{d_1}$, $\gamma \approx \tan\gamma = \frac{h}{d_2}$

3.9 Cont'd

$$\Rightarrow \alpha = \beta+\gamma = \frac{h}{d_1}+\frac{h}{d_2} = h\left(\frac{d_1+d_2}{d_1\cdot d_2}\right)$$

$$\Rightarrow h = \alpha\cdot\frac{d_1\cdot d_2}{d_1+d_2}$$

$$\Rightarrow \nu = h\cdot\sqrt{\frac{2}{\lambda}\cdot\frac{d_1+d_2}{d_1\cdot d_2}} = \alpha\cdot\frac{d_1 d_2}{d_1+d_2}\cdot\sqrt{\frac{2}{\lambda}\cdot\frac{d_1+d_2}{d_1 d_2}}$$

$$= \alpha\cdot\sqrt{\frac{2d_1 d_2}{\lambda(d_1+d_2)}}$$

$\boxed{3.10}$ Given $P_t = 10W$, $G_t = 10dB = 10$, $L = 1dB \doteq 1.259$.

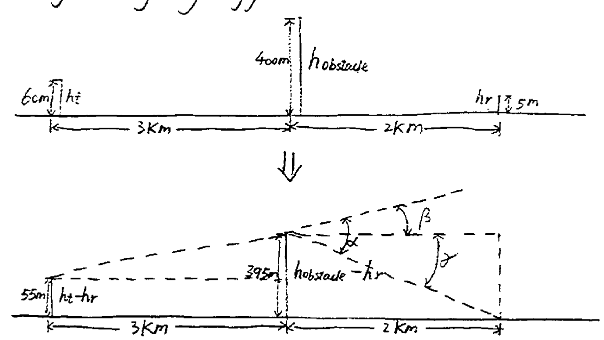
$G_r = 3dB = 2$, $f_c = 900MHz$, $d = 3000+2000 = 5000m$,

We have $\lambda_c = \frac{c}{f_c} \doteq 0.333(m)$ and free space received power

$$P_{freespace} = \frac{P_t\cdot G_t\cdot G_r\cdot\lambda_c^2}{(4\pi)^2 d^2\cdot L} = \frac{10\times10\times2\times0.333^2}{(4\pi)^2\times(5000)^2\times1.259}$$

$$\doteq 4.48\times10^{-9}(W) \doteq \underline{\underline{-53.5\ dBm}}$$

For the geometry shown below, we can redraw it in another geometry by approximation.



From the figure above we have

3.10 Cont'd

$\tan\beta = \frac{h_{obstacle} - h_t}{d_1} = \frac{400-60}{3000} \doteq 0.1133 \Rightarrow \beta \doteq 0.11285\ (rad)$

$\tan\gamma = \frac{h_{obstacle} - h_r}{d_2} = \frac{400-5}{2000} = 0.1975 \Rightarrow \gamma \doteq 0.1950\ (rad)$

$\Rightarrow \alpha = \beta + \gamma = 0.11285 + 0.195 \doteq 0.3078\ (rad)$

and $\nu = \alpha \cdot \sqrt{\frac{2 d_1 d_2}{\lambda (d_1+d_2)}} = 0.3078 \times \sqrt{\frac{2\times 3000 \times 2000}{0.333 \times (3000+2000)}} \doteq 26.12$

Using the approximation equation (3.59.e), we obtain

$G_d(dB) = 20 \cdot \log_{10}\left(\frac{0.225}{\nu}\right) \quad \nu > 2.4$

$= 20 \cdot \log_{10}\left(\frac{0.225}{26.12}\right)$

$\doteq -41.3\ dB.$

$\Rightarrow P_{received} = P_{freespace}(dBm) + G_d$

$= -53.5 - 41.3$

$= \underline{\underline{-94.8\ dBm}}$

$\Rightarrow$ loss due to diffraction $L_d = P_{freespace} - P_{received} = \underline{\underline{41.3\ dB}}$

$\boxed{3.11}$ (a) $f_c = 50\ MHz \Rightarrow \lambda_c = \frac{c}{f} = 6\ m$

$\Rightarrow P_{freespace} = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda_c^2}{(4\pi)^2 \cdot d^2 \cdot L} \doteq 1.45 \times 10^{-6} (W) \doteq -28.4\ (dBm)$

$\nu = \alpha \cdot \sqrt{\frac{2 d_1 d_2}{\lambda_c (d_1+d_2)}} = 0.3078 \times \sqrt{\frac{2\times 3000 \times 2000}{6 \times (3000+2000)}} \doteq 6.156$

$\Rightarrow G_d = 20 \log_{10}\left(\frac{0.225}{\nu}\right) = 20 \log_{10}\left(\frac{0.225}{6.156}\right) \doteq -28.7\ dB$

$\Rightarrow P_{received} = P_{freespace} + G_d = -28.4 - 28.7 = \underline{\underline{-57.1\ dBm}}$

$L_d = -G_d = \underline{\underline{31.7\ dB}}$

3.11 Cont'd

(b) $f_c = 1900\ MHz \Rightarrow \lambda = \frac{c}{f_c} \doteq 0.158\ (m)$

$\Rightarrow P_{freespace} = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda_c^2}{(4\pi)^2 \cdot d^2 \cdot L} \doteq 10^{-9} (W) = -60\ (dBm)$

$\nu = \alpha \cdot \sqrt{\frac{2 d_1 \cdot d_2}{\lambda_c (d_1+d_2)}} = 0.3078 \cdot \sqrt{\frac{2\times 3000 \times 2000}{0.158 \times 5000}} \doteq 37.9$

$\Rightarrow G_d = 20 \log\left(\frac{0.225}{\nu}\right) = 20 \cdot \log\left(\frac{0.225}{37.9}\right) = -44.5\ dB$

$\Rightarrow P_{received} = P_{freespace} + G_d = -60 - 44.5 = \underline{\underline{-104.5\ dBm}}$

$L_d = -G_d = \underline{\underline{44.5\ dB}}$

$\boxed{3.12}$ $n = -3.5 \Rightarrow \overline{P_r(d)}\ (dBm) = -35 \log_{10}\frac{d}{d_0} + P_r(d_0)(dBm)$

$= -35 \cdot \log_{10}\frac{10}{1} + 0 = -35\ dBm$

For $\gamma = -25\ dBm$, $Pr[P_r(d) > \gamma] = Q\left(\frac{\gamma - \overline{P_r(d)}}{\sigma}\right) = 10\%$

$\Rightarrow Q\left(\frac{-25-(-35)}{\sigma}\right) = Q\left(\frac{10}{\sigma}\right) = 10\%$

$\Rightarrow \frac{10}{\sigma} \doteq 1.29 \Rightarrow \sigma \doteq \underline{\underline{7.75\ dB}}$

$\boxed{3.13}$ (a) <u>For free space</u>, $P_r = P_0 \left(\frac{d_0}{d}\right)^2$

Given $P_0 = 10^{-6} (W) = -30\ dBm$, $d_0 = 1\ Km$.

<u>For $d = 2\ Km$</u>, $P_r = 10^{-6} \cdot \left(\frac{1}{2}\right)^2 = 2.5 \times 10^{-7} (W) \doteq \underline{\underline{-36\ dBm}}$

Similarly, <u>For $d = 5\ Km$</u>, $P_r \doteq \underline{\underline{-44\ dBm}}$

<u>For $d = 10\ Km$</u>, $P_r \doteq \underline{\underline{-50\ dBm}}$

<u>For $d = 20\ Km$</u>, $P_r = \underline{\underline{-56\ dBm}}$

(b) <u>For $n = 3$</u>, $P_r = P_0 \cdot \left(\frac{d_0}{d}\right)^3$