Problem 3.24

1) Since $\mathcal{F}[\text{sinc}(400t)] = \frac{1}{400}\Pi(\frac{f}{400})$, the bandwidth of the message signal is $W = 200$ and the resulting modulation index

$$\beta_f = \frac{k_f \max[|m(t)|]}{W} = \frac{k_f 10}{W} = 6 \Longrightarrow k_f = 120$$

Hence, the modulated signal is

$$\begin{aligned} u(t) &= A\cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau) \\ &= 100\cos(2\pi f_c t + +2\pi 1200 \int_{-\infty}^{t} \text{sinc}(400\tau)d\tau) \end{aligned}$$

2) The maximum frequency deviation of the modulated signal is

$$\Delta f_{\max} = \beta_f W = 6 \times 200 = 1200$$

3) Since the modulated signal is essentially a sinusoidal signal with amplitude $A = 100$, we have

$$P = \frac{A^2}{2} = 5000$$

4) Using Carson's rule, the effective bandwidth of the modulated signal can be approximated by

$$B_c = 2(\beta_f + 1)W = 2(6+1)200 = 2800 \text{ Hz}$$

Problem 3.26

1) Since an angle modulated signal is essentially a sinusoidal signal with constant amplitude, we have

$$P = \frac{A_c^2}{2} \Longrightarrow P = \frac{100^2}{2} = 5000$$

The same result is obtained if we use the expansion

$$u(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos(2\pi(f_c + nf_m)t)$$

along with the identity

$$J_0^2(\beta) + 2\sum_{n=1}^{\infty} J_n^2(\beta) = 1$$

2) The maximum phase deviation is

$$\Delta\phi_{\max} = \max|4\sin(2000\pi t)| = 4$$

3) The instantaneous frequency is

$$\begin{aligned} f_i &= f_c + \frac{1}{2\pi}\frac{d}{dt}\phi(t) \\ &= f_c + \frac{4}{2\pi}\cos(2000\pi t)2000\pi = f_c + 4000\cos(2000\pi t) \end{aligned}$$

Hence, the maximum frequency deviation is

$$\Delta f_{\max} = \max|f_i - f_c| = 4000$$

4) The angle modulated signal can be interpreted both as a PM and an FM signal. It is a PM signal with phase deviation constant $k_p = 4$ and message signal $m(t) = \sin(2000\pi t)$ and it is an FM signal with frequency deviation constant $k_f = 4000$ and message signal $m(t) = \cos(2000\pi t)$.