Solutions Chapter 11

1\. a) $z(w) = A\int^{w}(w-u_1)^{((\alpha_1/\pi)-1)}dw + B$

Take B=0 and the lower limit of the integral as 0. In this way w=0 has image z=0. The above reduces to

$z(w) = A\int_0^{w}(w)^{(\alpha_1/\pi)-1)}dw == Aw^{\alpha/\pi}$. To make z=1 have image w=1 we take A=1. Thus

$z(w) = w^{\alpha/\pi}$ and $w(z) = z^{\pi/\alpha}$ where the principal branches of the functions are used as the branch cuts do not extend into the domains of interest.

b) $\Phi(w) = w \;\; \Phi(w(z)) = z^{\pi/\alpha}$. Thus $V = \overline{\left(\frac{d\Phi}{dz}\right)} = \frac{\pi}{\alpha}\overline{\left(z^{\pi/\alpha-1}\right)}$

Since $0 < \alpha < \pi$ we have $\pi/\alpha - 1 > 0$. With $\alpha = \pi/4, V = 4\overline{\left(z^3\right)}$. Thus, the velocity in the z plane is infinite at infinity but nowhere else. It is zero in the corner where $z = 0$.

c)
```
test=1
while test <100

z=input('z=')
vel=4*conj(z^3)
test=input ('test =')
end
```

z=exp(i*pi/5)

vel =

-1.2361 - 3.8042i

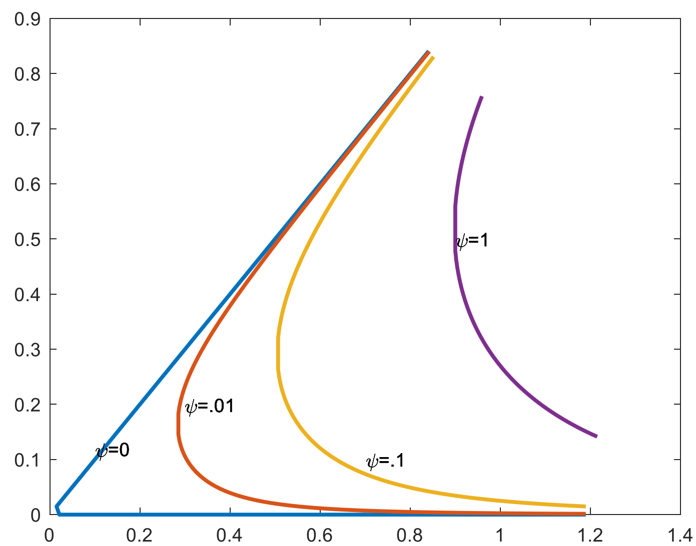
z=.1*exp(i*pi/8)

0.0015 - 0.0037i

d)

```
clf
clear
psi=[ 0 .01 .1 1 ];
u=linspace(-2,2,10^7);
for j=1:length(psi)
```

```
 w=u+i*psi(j);
 PHI=w.^(1/4);
 plot(real(PHI),imag(PHI),'linewidth',2);hold on
 text(.1,.12,'\psi=0')
 text(.3,.2,'\psi=.01')
 text(.7,.1,'\psi=.1')
 text(.90,.5,'\psi=1')
end
```

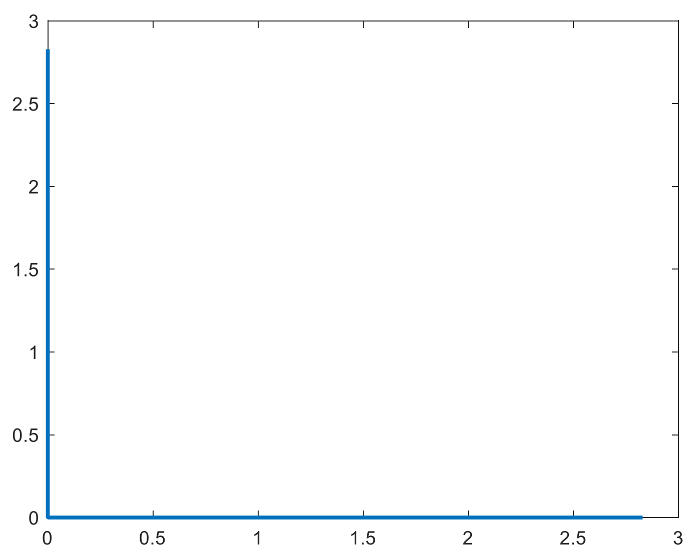


e) $z(w) = A\int^{w} (w)^{((\frac{2\pi}{\pi}-\frac{\alpha}{\pi})-1)} dw + B$. Take B =0 as in part a).Thus

$z(w) = A\int_0^{w} (w)^{(-\frac{\alpha}{\pi}+1)} dw = \dfrac{Aw^{2-\alpha/\pi}}{2-\alpha/\pi}$. Now to map $w=-1$ into $z=1$ we take $1 = \dfrac{A(-1)^{2-\alpha/\pi}}{2-\alpha/\pi} = \dfrac{A(-1)^{-\alpha/\pi}}{2-\alpha/\pi}$ and so $A = (-1)^{\alpha/\pi}(2-\alpha/\pi)$. Thus $z = (-1)^{\alpha/\pi} w^{2-\alpha/\pi}$

f) With $\alpha = \pi/2$ we have $z = (-1)^{1/2} w^{3/2}$.

```
w=linspace(-2,2,100);
z=(-1)^(1/2) *w.^(3/2);
plot(z,'linewidth',2);hold on
```



$\Phi(w) = w$, $z = (-1)^{1/2} w^{3/2} = i\left(w^{1/2}\right)^3$, If w is negative real then z is positive real, if we take $(-1)^{1/2} = i$ and use the principal value of $w^{1/2}$. If w is positive real then $z = i(|w|)^{3/2}$ which is positive imaginary. Now $-z^2 = w^3$ and so $w = (-1)^{1/3} z^{2/3}$. Let us take the branch cut of this function along the **positive** real z axis. We cannot have a branch cut in any domain lying in $\pi/2 < \arg(z) < 3\pi/2$. MATLAB will evaluate $(-z)^{2/3}$ as a principal value which means that the branch cut for this function lies along the positive real z axis since this is where -z is negative real.

$z^2 = -w^3 \quad \mathrm{w} = (-1)^{1/3} z^{2/3}$

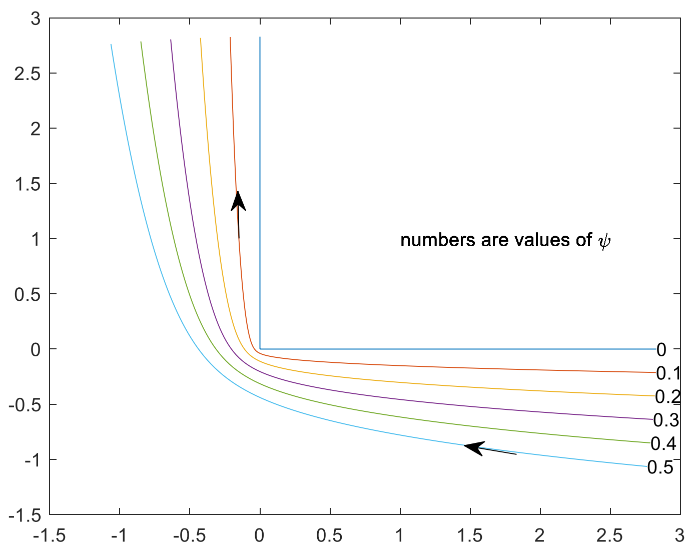
$\Phi(z) = (-1)^{1/3}(z)^{2/3} \quad \dfrac{d\Phi}{dz} = (-1)^{1/3}\dfrac{2}{3}z^{-1/3} = \dfrac{2}{3}\left(\dfrac{-1}{z}\right)^{1/3}$. If z is a positive real, we want this result to be real. Since MATLAB uses principal values, the angle in the resulting expression for $(-z)^{-1/3}$ will be $-2\pi/3$. However, any value of $(-z)^{-2/3}$ can be converted to another possible value by multiplying it by $e^{\pm i2\pi/3}$. Thus, we will take all values obtained for $(-z)^{-2/3}$ in MATLAB and multiply them by $e^{i2\pi/3}$. In this way if z is positive real then the expression for $(-z)^{-1/3}$ will be a negative real. If z positive real , then the velocity will be in the negative z direction. Now if z is positive imaginary the velocity should be a positive imaginary number. This is verified in the following program

```
test=1
while test <100

z=input('z=')
vel=2/3*conj(((-z)^(-1/3))*exp(-i*2*pi/3))
test=input ('test =')
end
```

Streamlines

$\Phi(w) = w = u + iv$. The streamlines are the lines in the w plane on which v is constant. We can map them into the streamlines in the z plane . We will take v= .1 .2 .3 .4 and .5 for ψ and use $z = (-1)^{1/2} w^{3/2}$ for the mapping , using i for sqrt(-1).

```
u=linspace(-2,2,100);
v=[0 .1 .2 .3 .4 .5];
clf
for j=1:length (v)
    w=u+i*v(j);

    z=i*(w.^(3/2))
    plot(z);hold on
    Psi=num2str(v(j))
    text(real(z(1)),imag(z(1)),Psi)
    text(1,1,'numbers are values of \psi')
end
```



2. The transformation z(w) is the same as in Example 1 and is $z(w) = \frac{-1}{\pi}\log(w + (w^2-1)^{1/2}) + i$ and once again we have $w = -\cosh(\pi z)$. The boundary conditions in the w plane are now different and we have on the line v=0, $-\infty < u < 1,\ \phi = 0$ and for $u > 1$ that $\phi(u,0)=1$. We again find $\phi(u,v) = \frac{v}{\pi}\int_1^{\infty}\frac{du'}{(u'-u)^2+v^2}$ which we evaluate as

result =

(v*(atan((u - 1)/v)/v + pi/(2*v*sign(v))))/pi

>> simplify(result)

ans =

(sign(v)*(pi + 2*atan((u - 1)/v)*sign(v)))/(2*pi)

Since v>0 the above is

$$\phi(u,v) = \frac{1}{2} + \frac{1}{\pi}\arctan\frac{u-1}{v}$$

This is $\phi(u,v)=\frac{1}{2}+\frac{1}{\pi}\arctan\frac{u-1}{v}=\frac{1}{2}+\frac{1}{\pi}\left(\frac{\pi}{2}-\arctan\frac{v}{u-1}\right)=1-\frac{1}{\pi}\left(\arctan\frac{v}{u-1}\right)$

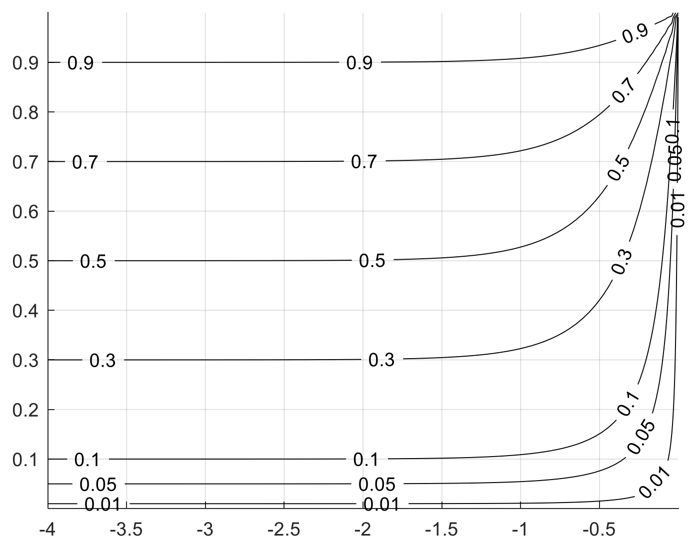
Notice that the above is the real part of the complex potential $1+\frac{i}{\pi}\mathrm{Log}(w-1)$.

Thus, in the z plane the complex potential is $1+\frac{i}{\pi}\mathrm{Log}(-\cosh(\pi z)-1)=\Phi(z)=\frac{i}{\pi}\mathrm{Log}(\cosh(\pi z)+1)$

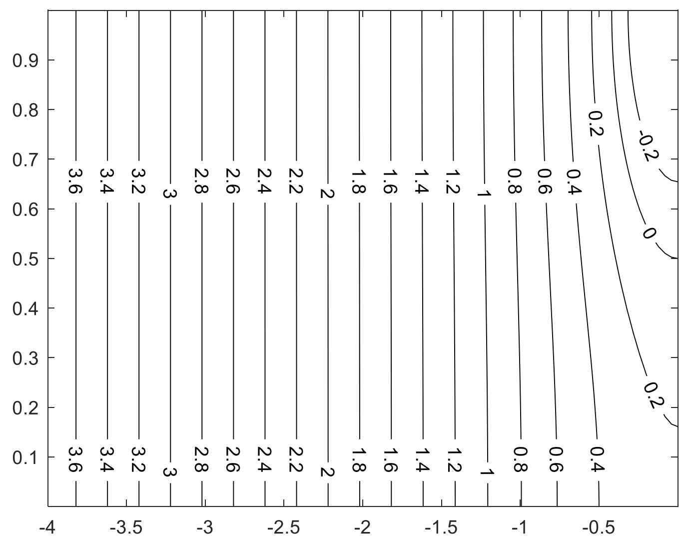
The code for generating the 2 plots is

```
x=linspace(-4,-10*eps,100);
y=linspace (10*eps,1-10*eps,100);
[X Y]=meshgrid(x,y);
z=X+i*Y;
Phi=i/pi*log((cosh(pi*z)+1));
phi=real(Phi);
figure(1)
hold on
[c,h]=contour (X,Y,phi,[.01 .05 .1 .3 .5 .7 .9])
clabel(c,h);colormap([0 0 0])
grid
psi=imag(Phi);
figure(2)
n=-1:30;
n=n*.2;
n=[0 n];
[p,q]=contour (X,Y,psi,n);colormap([0 0 0])
clabel(p,q)
hold on
```

And the results are



Equipotentials



Streamlines

3. a) and b)

$z_1 = i$ has image $w_1 = -1$ and $z_2 = 0$ has image $w_2 = 0$

We have to do this integration $z(w) = A\int^{w} (w)^{1/2}(w+1)^{1/2}\,dw + B$.

Since the lower limit of the integral is taken as zero we can take B=0.

We take the lower limit of integration as 0 and put B=0. In this way the point $w = u_1 = 0$ is mapped into $z_1 = 0$.From MATLAB we can obtain for the indefinite integral.

The following code does the above integration , employs the lower limit of integration and also shows how to get A.

```
syms w W
clf
f=@(w)w^(1/2)*(w+1)^(1/2)
g=int(f,w)
format long
```

```
% second part uses g=I found above
I=@(w)w^(1/2)*(w/2 + 1/4)*(w + 1)^(1/2)- log(w +…
 w^(1/2)*(w + 1)^(1/2) + 1/2)/8
format long
 lowerlim=I(0)

%the following ensures that w=0 has image z=0
Inew=@(w)w^(1/2)*(w/2 + 1/4)*(w + 1)^(1/2) - log(w+…
w^(1/2)*(w + 1)^(1/2) + 1/2)/8-I(0)

%the following ensures that w=-1 has image z=i
A=i/Inew(-1)
z=@(w)A*(w.^(1/2).*(w./2 + 1/4).*(w + 1).^(1/2-log(w+…
w.^(1/2).*(w + 1).^(1/2) + 1/2)/8)-A*I(0)

%the following tests that the real axis in the w plane is
mapped onto
%the U shaped boundary in the z plane
checkone=z(-1)
check2=z(0)
w=linspace(-3,3,100);
Z=z(w);
plot(real(Z),imag(Z),'linewidth',3); hold
```

—

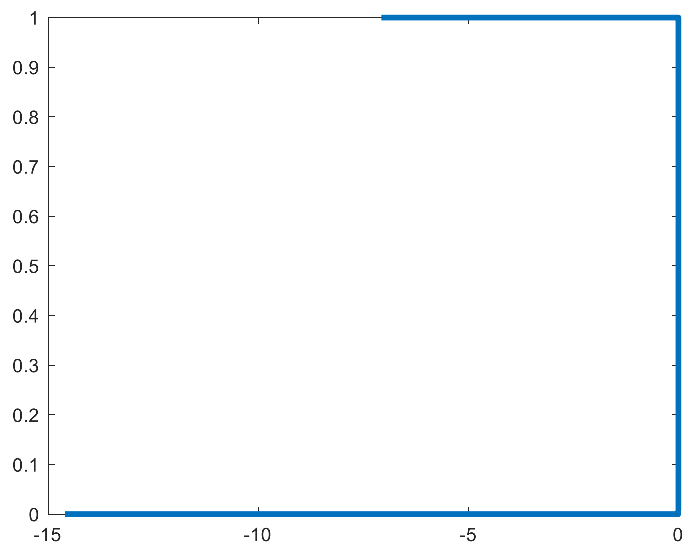
This is the output which provides a check on our result

A =

 -2.546479089470326

lowerlim =

  0.086643397569993



c)The image of the vertical portion of the boundary in the z plane is the line segment in the w plane v=0, $-1 < u < 0$. This is maintained at 1 volt while the remainder of this line is at u volts. From the Poisson integral formula for the upper half plane we have that the voltage in the upper half of the w plane is given by $\phi(u,v) = \frac{v}{\pi}\int_{-1}^{0}\frac{du'}{(u'-u)^2+v^2}$

From MATLAB

```
syms up u v
assume(u,'real')
assume(v,'real')
f=1./((up-u)^2+v.^2);
result=int(f,up,[-1  0]);
result=v/pi*result;
pretty (result)

the output is
     / u \       / u + 1 \
  atan| - | - atan| ----- |
      \ v /       \   v   /
- -------------------------
              pi
```

Which is the same as

$$\frac{1}{\pi}\left(\arctan(v/u)-\arctan(v/(u+1))\right)=\frac{1}{\pi}\arctan\frac{(v/u-v/(u+1))}{1+\dfrac{v^2}{u(u+1)}}=\frac{1}{\pi}\arctan\frac{v}{u^2+u+v^2}.$$

Suppose we want the locus of a voltage V. Then we have $\tan(\pi V)=K=\dfrac{v}{u^2+u+v^2}$ which is the equation of circles. We can rewrite this as $(u+1/2)^2+(v-1/(2k))^2=\frac{1}{4}(1+\left(\frac{1}{k}\right)^2)$. As a check can take V=1/2. Then $k=\infty$. The circle has center at $u=-1/2, v=0$ and radius $1/2$. Notice that in general all circles are centered at $u=-1/2$ $v=1/(2k)$ where $k=\tan(\pi V)$. We can plot these circles using this code

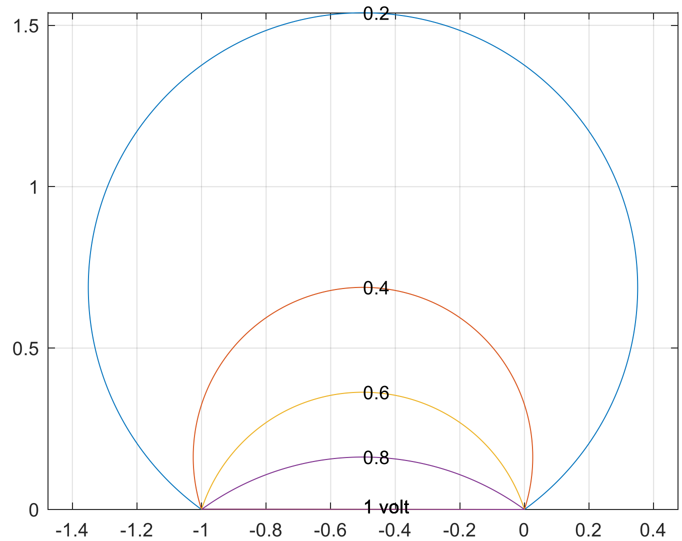
```
V=input('V=')

syms w
while V<1
    clf

k=tan(pi*V);
alpha=asin(1/( sqrt(k^2+1)))


psi=linspace(-alpha,pi+alpha,100);
r=(1/2)*sqrt(1+1/k^2);
w=-1/2+i/(2*k)+r*exp(i*psi);
plot(w);hold on;grid
V=input('V=')

end
```

Using the transformation found in parts a and b we can map any of the equipotentials found in the preceding code into the z plane from the w plane

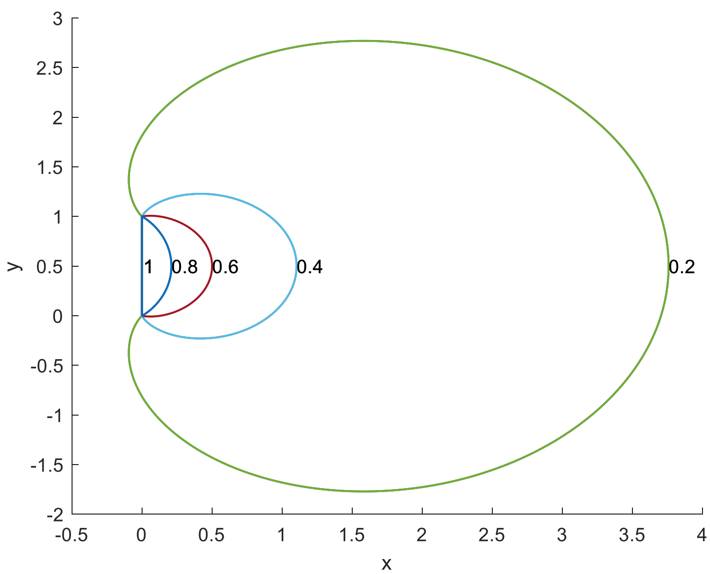
c)

```
V=1:4;
V=2*V/10;
%following is from preceding part of problem
 A= -2.546479089470326;
 lowerlim = 0.086643397569993;
clf
for j=1:length(V)
```

```
   Vo=V(j);
   voltlabel=num2str(V(j));
k=tan(pi*Vo);
alpha=asin(1/(sqrt(k^2+1)));
psi=linspace(-alpha,pi+alpha,1000);

r=(1/2)*sqrt(1+1/k^2);

w=-1/2+i/(2*k)+r*exp(i*psi);
wlabel=-1/2+i/(2*k)+r*exp(i*pi/2);
xlabel('u'); ylabel('v');
RR=imag (w)>0;
w=w.*RR;
figure(1)
plot(w);hold on; axis equal
text(real(wlabel),imag(wlabel),voltlabel)
text(-.5,.01,'1 volt')


format long

I=@(w)w^(1/2)*(w/2 + 1/4)*(w + 1)^(1/2) - log(w +
w^(1/2)*(w + 1)^(1/2) + 1/2)/8;

z=@(w)A*(w.^(1/2).*(w./2 + 1/4).*(w + 1).^(1/2) - log(w+
w.^(1/2).*(w + 1).^(1/2) + 1/2)/8)-A*lowerlim;

 Z=z(w);
```



0.2
0.4
0.6
0.8
1 volt



1
0.8
0.6
0.4
0.2
x
y

d) From part c) we have

$$\phi(u,v) = \frac{1}{\pi}\left(\arctan(v/u) - \arctan(v/(u+1))\right)$$

Thus the potential is the real part of the complex potential

$\Phi(w) = \frac{-i}{\pi}\left[\text{Log}\, w - \text{Log}(w+1)\right] = \phi(u,v) + i\psi(u,v)$. The stream function is the imaginary part:

$$\psi(w) = \frac{1}{\pi}\left[\text{Log}\left|(w+1)\right| - \text{Log}\left|w\right|\right] = \frac{1}{\pi}\left[\text{Log}\left|\frac{(w+1)}{w}\right|\right]$$

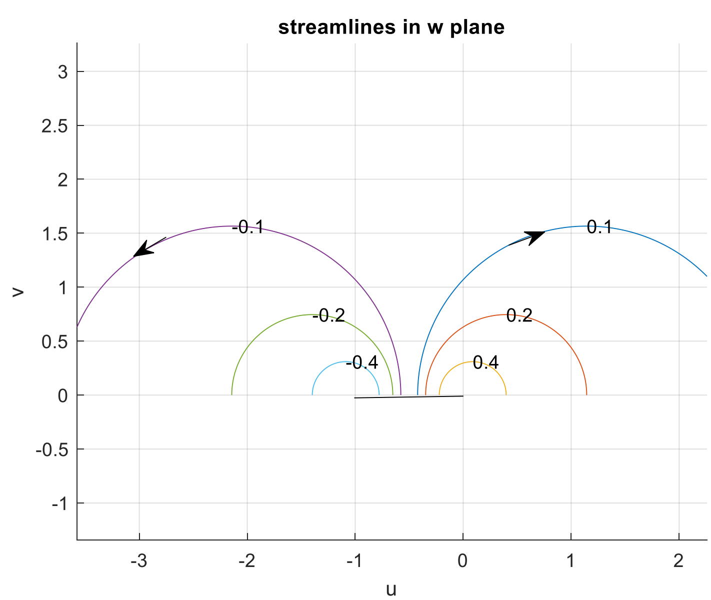
Thus $e^{\pi\psi} = k = \left|\frac{(w+1)}{(w)}\right|$ describes the streamlines.

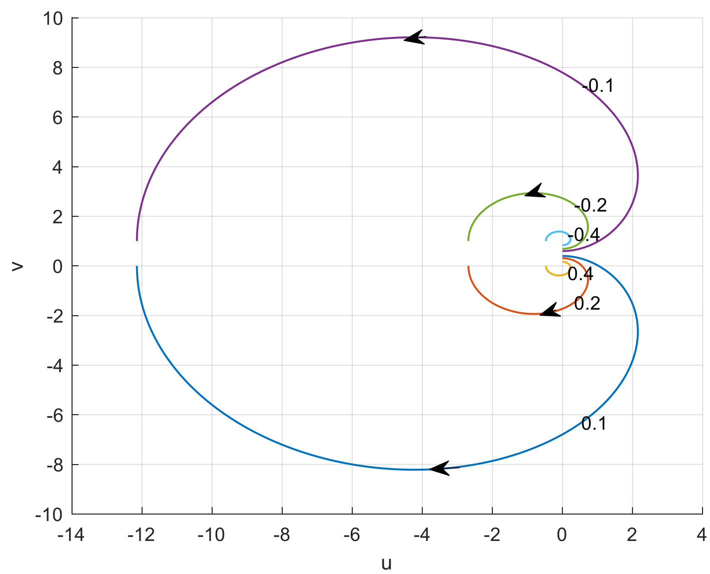
Since ψ is real it follows that $k \geq 0$ is real. With $w = u + iv$ the preceding becomes

$\left(u - \frac{1}{k^2-1}\right)^2 + v^2 = \frac{k^2}{\left(k^2-1\right)^2}$. Circles of radius $k/\left|(k^2-1)\right|$ center at v=0, $\frac{1}{k^2-1}$.

```
clear
psi=[.1 .2 .4 -.1 -.2 -.4]
%following is from preceding part of problem
 A= -2.546479089470326;
 lowerlim = 0.086643397569993;
clf
for j=1:length(psi)
    PSI=psi(j);
   psilabel=num2str(PSI);
k=exp(pi*PSI);

theta=linspace(0,pi,1000);

r=abs(k)/abs(k^2-1);
w=1/(k^2-1)+r*exp(i*theta);
xlabel('u'); ylabel('v');
RR=imag (w)>0;
figure(1);hold on
plot(w); axis equal
val_psi=num2str(PSI);
text(1/(k^2-1),r,val_psi);
format long
```

```
I=@(w)w^(1/2)*(w/2 + 1/4)*(w + 1)^(1/2) - log(w +
w^(1/2)*(w + 1)^(1/2) + 1/2)/8;
 z=@(w)A*(w.^(1/2).*(w./2 + 1/4).*(w + 1).^(1/2) - log(w+
w.^(1/2).*(w + 1).^(1/2) + 1/2)/8)-A*lowerlim;
 Z=z(w);
Z_for_label=z((1/k^2-1)+i*r)
figure(2);hold
plot(Z,'linewidth',1); hold ;
text(real(Z(500)),imag(Z(500)),val_psi)
end
figure(1);grid; hold on
figure(2);grid
```





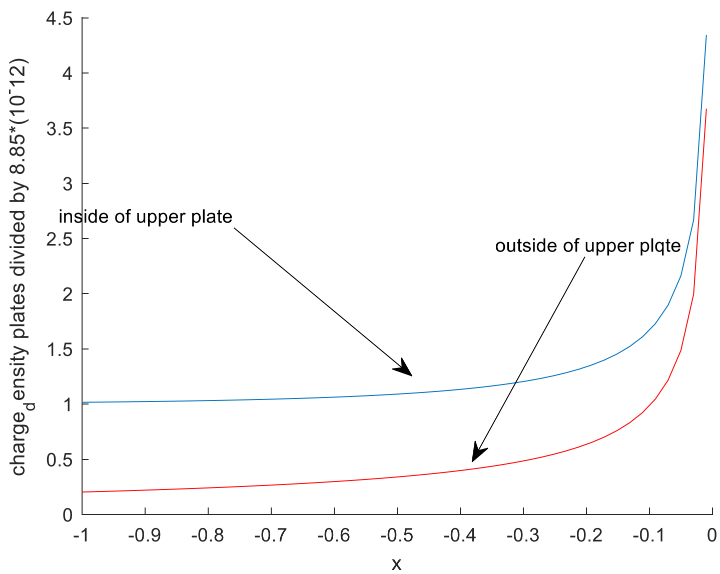
Streamlines in the z plane

Note the negative signs in front of numbers in upper half plane of z plane. They are hard to see.

4.

```
clear
%solution for parts a and b
x=linspace(-1,-.01,50);
clf
for j=1:length(x)
 z(j)=x(j)+.9999*i  ;
syms w
warning('off')

w=solve(1./pi*(w+log(w)+1) ==z(j));
```

```
E_field(j)=-i*w/(abs(w^2)+w);
result=[z(j), -E_field(j)];
hold on
end
figure(1)
plot(x,imag(-E_field)); hold on
xlabel('x');
ylabel('charge_density on inside of upper plate divided by 8.85*(10^-12)')
clear
%______________
x=linspace(-1,-.01,50);

for j=1:length(x)
 z(j)=x(j)+1.0001*i  ;
syms w
warning('off')

w=solve(1./pi*(w+log(w)+1) ==z(j));
E_field(j)=-i*w/(abs(w^2)+w);

hold on
end
figure(1)
plot(x,imag(E_field),'r'); hold on
xlabel('x');
ylabel('charge_density plates divided by 8.85*(10^-12)')
```



5.

a) $z=-\left(w^2-1\right)^{-1/2}=-(w-1)^{-1/2}(w+1)^{-1/2}$. Take note of the branches and their cuts
Suppose w is real and >1. Then the above expression is negative real and satisfies $-\infty<z<0-$. Suppose w is real and <-1. Then the above expression is positive real and satisfies $0+<z<\infty$. If w is real and $-1<w<1$ then this expression is positive imaginary and satisfies $1<\operatorname{Im} w<\infty$

b) The image of the positive imaginary axis in the w plane is the line segment $v=0,-1<u<1$, which is maintained at 1 volt. The remainder of that segment is maintained at 0 volts. This is the boundary condition to be satisfied by the $\phi=\operatorname{Re}\Phi(w)$. Using the Poisson integral formula for the upper half plane we see that $\phi(u,v)=\frac{v}{\pi}\int_{-1}^{1}\frac{du'}{(u'-u)^2+v^2}$..

This was evaluated in Example 1 and found to be

$$\phi(u,v)=\frac{\arctan(v/(u-1))-\arctan(v/(u+1))}{\pi}$$

From this we found the corresponding complex potential in the w plane to be

$\Phi(w)=\frac{-i}{\pi}\left[\operatorname{Log}(w-1)-\operatorname{Log}(w+1)\right]\quad w=u+iv$. The imaginary part of this expression is the stream function and given by $\psi(w)=\frac{1}{\pi}\left[\operatorname{Log}\left|\frac{w+1}{w-1}\right|\right]\quad w=u+iv$

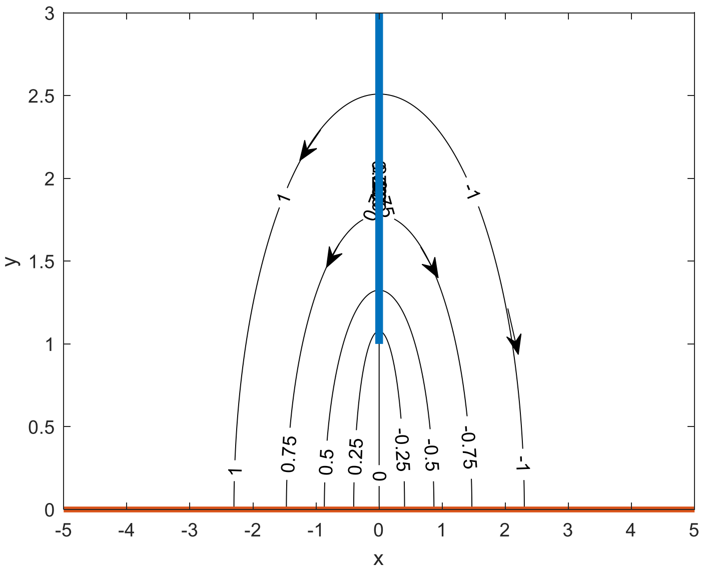
Solving $z=-(w-1)^{-1/2}(w+1)^{-1/2}$ for *w* we have $w=\frac{(1+z^2)^{1/2}}{z}$. Notice that if z is positive and greater than 1 that MATLAB would evaluate w as positive real. We want w to be negative real. Thus, we take in our code $w=-\frac{(1+z^2)^{1/2}}{z}$ Thus, in the z plane the stream function is

$$\psi(z)=\frac{1}{\pi}\left[\operatorname{Log}\left|\frac{-(1+z^2)^{1/2}+z}{-(1+z^2)^{1/2}-z}\right|\right]=\frac{1}{\pi}\left[\operatorname{Log}\left|\frac{-(1+z^2)^{1/2}+z}{(1+z^2)^{1/2}+z}\right|\right]$$

The following code will generate the streamlines, using **contour**

```
x=linspace (-5,5,500);
clf
y=linspace (0,3,10000);
[X Y]=meshgrid(x,y);
z=X+i*Y;
z1=-sqrt(z.^2+1);
z2=(z1+z)./(z1-z);
z2=abs(z2);
psi=1/pi*log(z2);
[c h]=contour(X,Y,psi,[.25 .5 .75 1 0 -.25 -.5 -.75 -1]);
colormap([0 0 0])
xlabel('x');ylabel('y')

clabel(c,h)
hold on
%the following generates the boundary including the barrier
y=linspace(1,3,100);
xx=eps*y;
%the following plots the barrier interrupting the flow.
 plot(xx,y,'linewidth',4)
hold on
yy=0*x;
plot(x,yy,'linewidth',3);
```

The output is



Some arrows were used to show direction of the streamlines.

c) The complex electrostatic potential in the w plane is

$$\Phi(w) = \frac{-i}{\pi}\left[\mathrm{Log}(w-1) - \mathrm{Log}(w+1)\right] \quad w = u + iv$$

The electric field vector in the z plane is given by

$$E = -\overline{\left(\frac{d\Phi}{dz}\right)} = -\overline{\frac{d\Phi}{dw}\frac{dw}{dz}} = \frac{-i}{\pi}\overline{\left[\frac{1}{w-1}-\frac{1}{w+1}\right]}\;\overline{\frac{d}{dz}\frac{(1+z^2)^{1/2}}{z}} = \frac{-i}{\pi}\overline{\left[\frac{1}{w-1}-\frac{1}{w+1}\right]}\,\overline{\left[\frac{-1}{z^2}(1+z^2)^{1/2}+(1+z^2)^{-1/2}\right]} =$$

$$\frac{i}{\pi}\overline{\left[\frac{2}{w^2-1}\right]}\,\overline{\left[\frac{1}{z^2(1+z^2)^{1/2}}\right]} = \frac{i}{\pi}2\overline{\left[\frac{1}{(1+z^2)^{1/2}}\right]}$$

As noted above, in MATLAB code we must take $(1+z^2)^{1/2} = -(1+z^2)^{1/2}$.

Commented [DW1]:

Code for electric field

```
k=1;
while(k>0)
x=input('x=')
y=input('y=')
z=x+i*y;
E=-i*2/pi*conj(1/(1+z^2)^(1/2))
k=input('k=')
end
```

```
At x=1, y=1 the electric field is
0.2238 - 0.3622i
Note that the field is directed downward and to the right.
At x=0,y=0+ the field is -2/pi  =-.6366 and is directed
downward.
```

6.
a) and b)

Moving from left to right along the boundary we encounter these angles (in the limit) $\alpha_1 = 3\pi/4, \alpha_2 = 2\pi, \alpha_3 = \pi/4$. Thus our transformation from Eq.(11.1) becomes

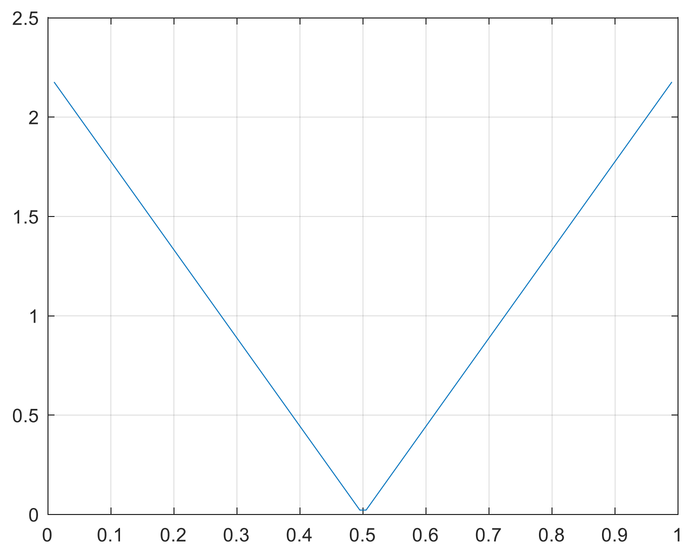
$$z(w) = A\int^{w}(w+1\ )^{-1/4}(w-u)\ (w-1)^{-3/4}dw + B$$

Taking the lower limit for the integration as -1 we see that taking *B*=0 ensures that w=-1 is mapped into z=0-.

Now we want w=1 to be mapped into z=0+. Thus we require $z(w) = \int_{-1}^{1}(w+1\ )^{-1/4}(w-u)\ (w-1)^{-3/4}dw = 0$. We can determine the value of u with this code:

```
clear
syms w W
 syms w
 u=linspace (.01,.99,100);
 for j=1:length(u)

my_integral(j)=integral(@(w)(((w+1).^(-1/4)).*(w-u(j)).*((w-1).^(-3/4))),-
1,1);
 end
 plot(u,abs(my_integral));grid
```

whose output is below and which shows that u=1/2.



c)

Finding A

```
clear
%Finding A
syms w W
 syms w
 up=1/2;
 clear
syms w
clf
up=1/2;
format long
A=exp(i*pi/4)/integral(@(w)(((w+1).^(-1/4)).*(w-up).*((w-
1).^(-3/4))),-1,.5)
```

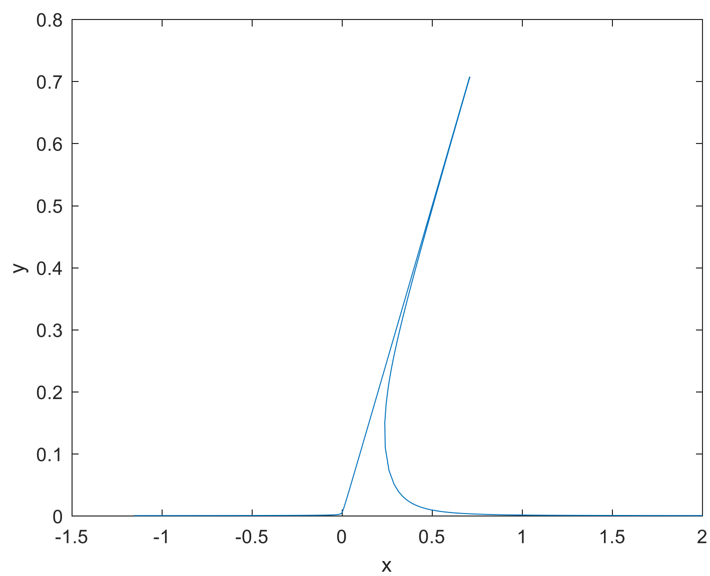
The output is

A= 0.877382647087441 - 0.000000000000000i

d)

```
clear;clf
syms w
u=linspace(-2,2,6000);
 W=u+.001*i;
 A=0.877382647087441;
 for j=1:length(W);
 my_integral(j)=A*integral(@(w)(((w+1).^(-1/4)).*(w-
.5).*((w-1).^(-3/4))),-1,W(j));
 end
z=my_integral;
plot(real(z),imag(z))
```

The output is



c) $\Phi(w) = Aw$ so that

$$\frac{d\Phi}{dz} = \frac{d\Phi}{dw}\frac{dw}{dz} = 1/(dz/dw) = \frac{A}{A}(w+1)^{1/4}(w-1)^{3/4}(w-1/2)^{-1} = (w+1)^{1/4}(w-1)^{3/4}(w-1/2)^{-1}$$

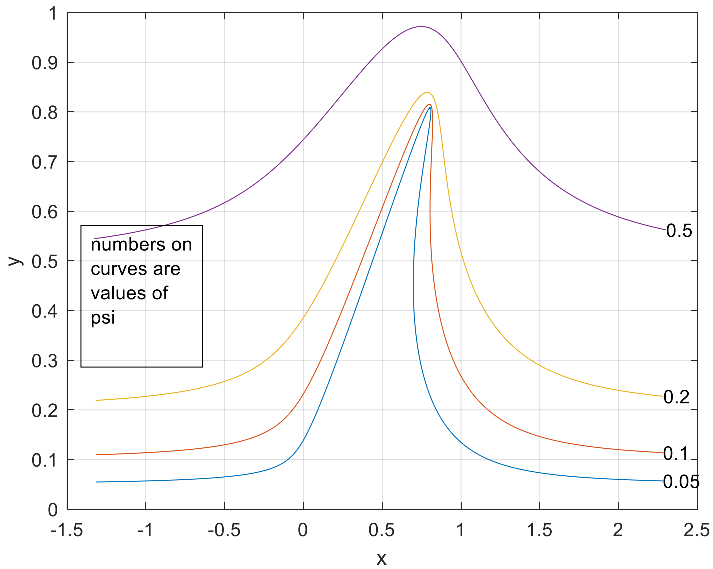
Note that as $w \to \infty$ this tends asymptotically to 1 using principal branches. Thus the velocity at infinity in the z plane is one.

d)

```
clear;clf
syms w
u=linspace(-2,2,6000);
v=[.02 .2 .5];
for k=1:length(v)
 W=u+v(k)*i;
 A=0.877382647087441;
 for j=1:length(u);
my_integral(j)=A*integral(@(w)(((w+1).^(-1/4)).*(w-
.5).*((w-1).^(-3/4))),-1,W(j));

 end
z=my_integral;
plot(real(z),imag(z));hold on
end
x=linspace(0,1/sqrt(2));
y=x;
plot(x,y,'linewidth',2)
```

e)

```
u=linspace(-2,2,6000);
clf
psi=[.05 .1 .2 .5];

 A=0.877382647087441;
 for k=1:length(psi)
 for j=1:length(u)
     W=u(j)+psi(k)*i;
 my_integral(j)=integral(@(w)(((w+1).^(-1/4)).*(w-.5).*((w-
1).^(-3/4))),-1,W);
 end
 z=my_integral;
 plot(real(z),imag(z)); hold on
 PSI=num2str(psi(k));
 L=length(u -10);
 text(real(z(L)),imag(z(L)),PSI)
 end
 grid
```



numbers on curves are values of psi
0.5
0.2
0.1
0.05
y
x