Lecture 7

Chapter 25

Electric Potential Energy

Course website:
https://sites.uml.edu/andriy-danylov/teaching/physics-ii/

Today we are going to discuss:

Chapter 25:

- Section 25.1. *Electric Potential Energy*
- Section 25.2. *The potential energy of a point charge*

New Idea

So far, we used vector quantities:

1. Electric Force (F) $F_{2\text{ on }1} = \dfrac{K|q_1||q_2|}{r^2}$

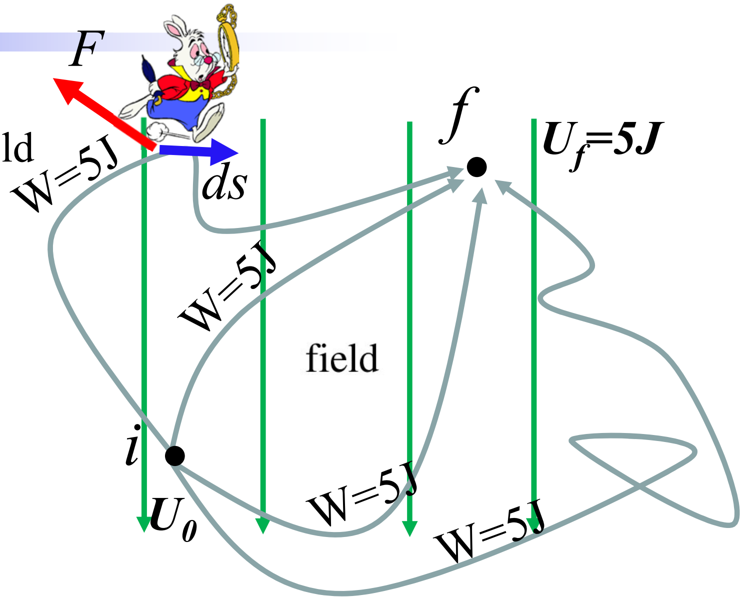
2. Electric Field (E) $\vec{\boldsymbol{E}} = \dfrac{\vec{\boldsymbol{F}}_{on\ q}}{\boldsymbol{q}}$

But, as you know, it is *not easy to deal with vectors*

Let's introduce *scalar quantities* instead of a FORCE and a FIELD

Let’s define a conservative force and give an idea of PE

Consider different paths between two points in a field



Recall for Physics I, a work done by a force

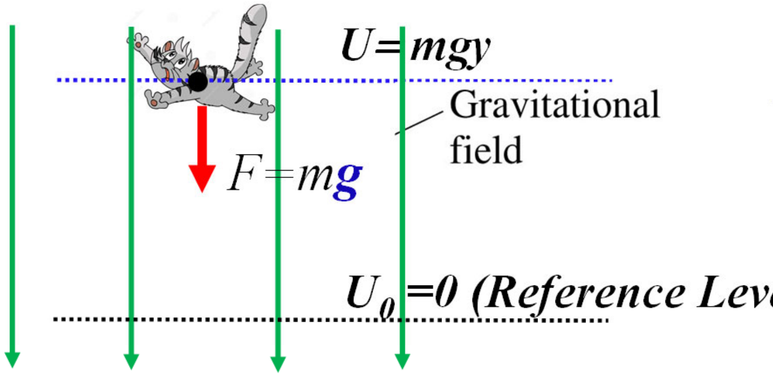
$$W = \int \vec{F} \cdot \overrightarrow{ds}$$

If a work done by a force is path independent, then this force is called *conservative* (since it leads to conservation of mechanical energy)

PE idea: That is what we have in our example(Lowell-Andower-5bucks). So, you can “attach” this number to the final point and give this number 5 a fancy name – the potential energy (PE) (U_f=5J) with respect to the initial point, where we can assume PE to be zero (the reference point).
For every conservative force, a potential energy can be introduced.

Since the gravitational and electrical forces are conservative forces, corresponding potential energies can be introduced.

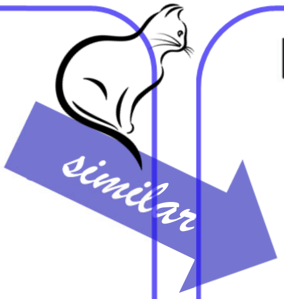
Gravitation



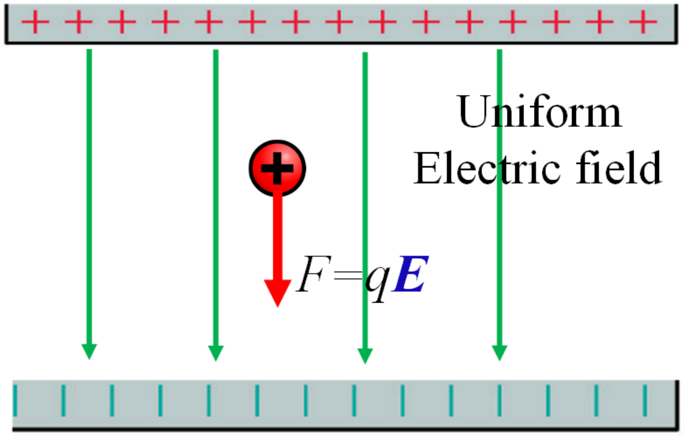
This fall can be described using a gravitational force (vector quantity), which describes an interaction between the Earth and the cat.

$$F_{grav} = mg \quad \Rightarrow \quad U_{grav} = mgy$$

Since the gravitational force is a conservative force, a gravitational potential energy (scalar quantity) can be introduced.



Electrostatics



Goal 1 is to introduce this:

$$F_{el} = qE \quad \Rightarrow \quad \boxed{U = qEs}$$

Let's derive these electric PE:

The case of two point masses

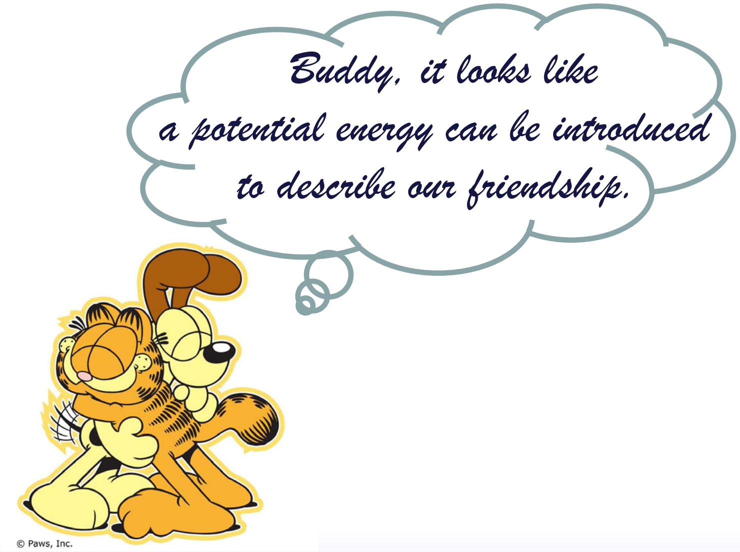
$$F_G = G\frac{m_1 m_2}{r^2} \xrightarrow{\text{Physics I}} U_G = G\frac{m_1 m_2}{r}$$

The case of two point charges

Goal 2 is to introduce this:

$$F_e = k\frac{q_1 q_2}{r^2} \quad \Rightarrow \quad \boxed{U(r) = \frac{kq_1 q_2}{r}}$$

Electric Potential Energy is an <u>energy of interaction</u>,
so there must be at least <u>two interacting</u> electric objects.



Potential energy of q in a uniform electric field (q in a capacitor)

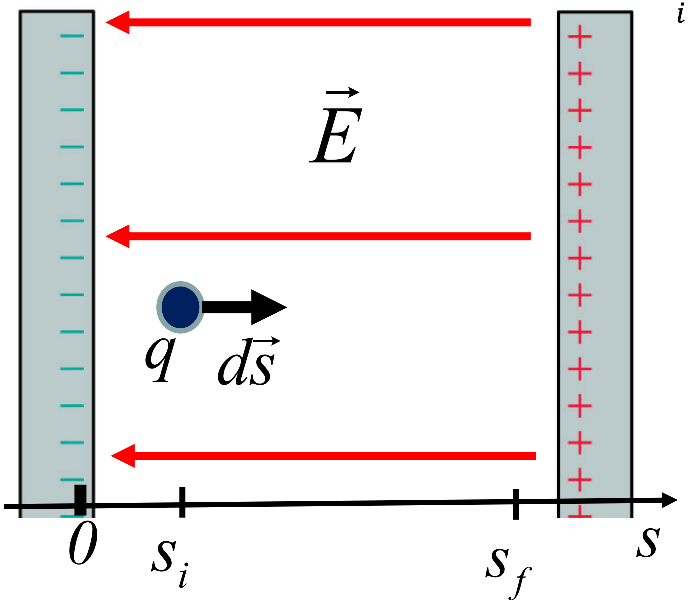
Potential energy of *q* in a uniform electric field

Consider a charge q inside a capacitor.

It moves from an initial point s_i to a final point s_f.
There is a constant force $F = qE$ acting on q

$\vec{E} \uparrow\downarrow d\vec{s}$ $\quad$ $E = const$

The work done on q is: $W = \int_i^f \vec{F} \bullet d\vec{s} = q \int_i^f \vec{E} \bullet d\vec{s} = -q \int_i^f E ds = -qE \int_i^f ds = -qE(s_f - s_i)$



Recall from Physics I $\boxed{-\Delta U = W = \Delta K}$

$$-\Delta U = -(U_f - U_i) = -qE(s_f - s_i)$$

$$U_f - U_i = qEs_f - qEs_i$$

$$U_f - U_i = qEs_f - qEs_i$$

Electric potential energy of charge q and a charged capacitor $\boxed{U = qEs}$

When you work with a potential energy, remember where your reference point/level is: here U_0=0 is at the negative electrode.

ConcepTest Potential energy

Two positive charges are equal. Which has more electric potential energy?

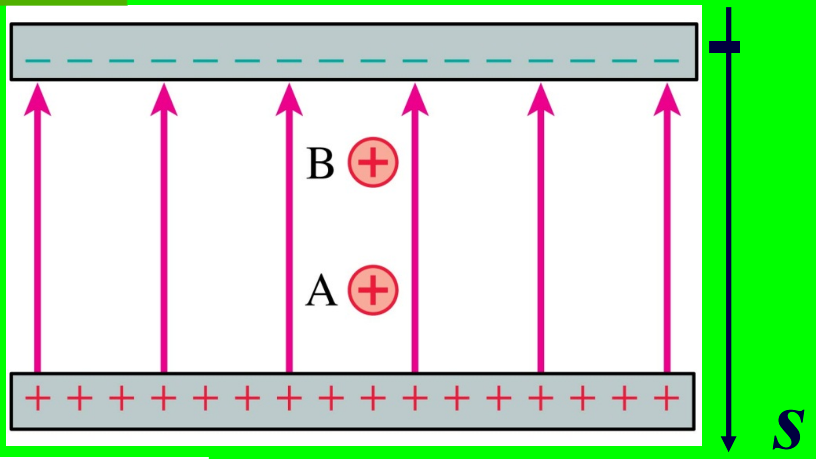
[✓ A) Charge A]

B) Charge B

C) They have the same potential energy

D) Both have zero potential energy

$$U = qEs$$



The potential energy of a positive charge decreases in the direction of $\vec{E}$.

ConcepTest Potential energy

Two negative charges are equal. Which has more electric potential energy?

A) Charge A

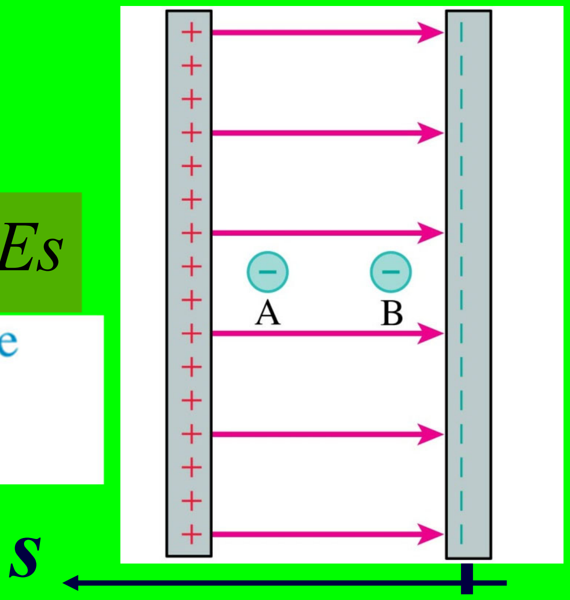
[✓ B) Charge B]

C) They have the same potential energy

D) Both have zero potential energy

$$U = -|q|Es$$

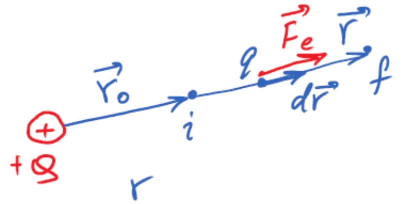
The potential energy of a negative charge decreases in the direction opposite to $\vec{E}$.



Potential energy of two point charges

The potential energy of two point charges

Electrostatic potential energy.



charge q moves in a field created by Q. Let's calculate work done by F_e as q moves from an initial to a final point:

$$W=\int_{r_0}^{r}\vec{F}_e\cdot d\vec{r}=\left\|\vec{F}_e\uparrow\uparrow d\vec{r}\right\|=\int_{r_0}^{r}F_e\,dr=\left\|F_e=k\frac{qQ}{r^2}\right\|=$$

$$=\int_{r_0}^{r}k\frac{qQ}{r^2}\,dr=-\frac{kqQ}{r}\Big|_{r_0}^{r}=-\left[\frac{kqQ}{r}-\frac{kqQ}{r_0}\right]$$

we know that $W=-\Delta U$; U – pot. energy, so

$$-\left[U(r)-U_0\right]=-\left[\frac{kqQ}{r}-\frac{kqQ}{r_0}\right]$$

As you remember, only differences in U have physical meaning. So we are free to choose any reference point. It's common to choose $U(r)$ to be zero at $r=\infty$.

Ref. point $r_0\to\infty$ $U_0=0$, so

$$U(r)-\cancel{U_0}^{\to 0}=\frac{kqQ}{r}-\cancel{\frac{kqQ}{r_0}}^{\to 0}$$

finally

$$\boxed{U(r)=k\frac{qQ}{r}}$$

pot. energy of the system of two charges Q, q.

$$\text{if}\ \left.\begin{matrix}q>0 & Q>0\\ q<0 & Q<0\end{matrix}\right\}\Rightarrow U(r)>0$$

$$\text{if}\ \left.\begin{matrix}q<0 & Q>0\\ q>0 & Q<0\end{matrix}\right\}\Rightarrow U(r)<0$$

$$\boxed{U(r)=\frac{kqQ}{r}}$$

This is explicitly the energy of the system, not the energy of just q or Q.

Note that the potential energy of two charged particles approaches zero as r → ∞.

ConcepTest Potential energy

A positive and a negative charge are released from rest in vacuum. They move toward each other. As they do:

A) A positive potential energy becomes more positive.

B) A positive potential energy becomes less positive.

[✓ C) A negative potential energy becomes more negative.]

D) A negative potential energy becomes less negative.

E) A positive potential energy becomes a negative potential energy.

$$U(r) = \frac{kqQ}{r}$$

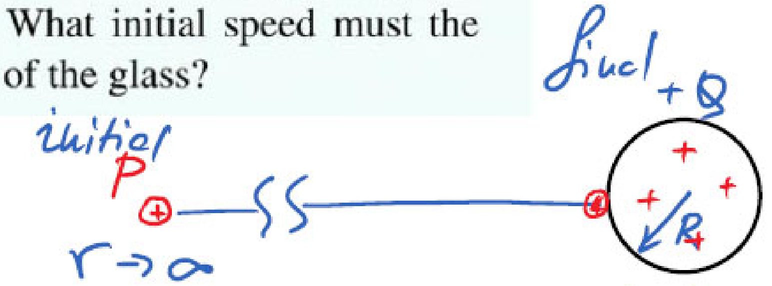
Opposite signs, so *U* is Negative.

U increases in magnitude as *r* decreases.

Example *Approaching a charged sphere*

A proton is fired from far away at a 1.0-mm-diameter glass sphere that has been charged to +100 nC. What initial speed must the proton have to just reach the surface of the glass?

Example 25.2



$U_i = \frac{kqQ}{r} = 0$

$K_i = \frac{1}{2} m v_0^2$;

$U_f = \frac{kqQ}{R}$

$K_f = 0$

it reaches with $v_f = 0$

Total energy is conserved, i.e

$$K_i + \cancel{U_i}^{\,0} = \cancel{K_f}^{\,0} + U_f$$

$$\frac{1}{2} m v_0^2 = \frac{kqQ}{R} \Rightarrow \boxed{v_0 = \sqrt{\frac{2kqQ}{m \cdot R}}} = 1.86 \cdot 10^7 \text{ m/s}$$


Thank you
Bye Bye For Now

ConcepTest

Potential energy

A positive charge moves as shown. Its kinetic energy

A) Increases.

B) Remains constant.

[✓ C) Decreases.]

The potential energy of a positive charge decreases in the direction of $\vec{E}$.

U increases $\Delta U > 0$

⇩ $-\Delta U = W = \Delta K$

Total energy is constant $-\Delta U = \Delta K$

⇩

$\Delta K < 0$

K decreases