MATH 5870-201 Measure and Probability Theory Spring 2024

Class meetings: Tuesday 6:30 PM – 9:20 PM Olsen 412

Instructor: Professor Alexander Kheifets

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Office Hours: Monday 3:30 – 5:30 p.m.
Tuesday 3:30 – 5:30 p.m.
And by appointment

Class Notes: Some class notes will be provided.

Supplemental Reading:

N. Vaillant, Probability Tutorials, available online at www.probability.net

The course objectives:

The goal of the course is to learn mathematical foundations of Probability Theory in terms of the Measure Theory. The following concepts of the Measure Theory will be: measure space, which is a triple of a set, sigma algebra of subsets and a measure (non-negative countably additive function on the sigma-algebra); functions measurable with respect to sigma-algebras and various types of convergence of sequences of such functions; Lebesgue integration. Measure Theory concepts will be given a Probability interpretation as probability spaces, random events, random variables, expectations and higher moments, distributions, independence. Precise mathematical statements and proofs will be given of some probability limit theorems such as Law of Large Numbers and the Central Limit Theorem. Those theorems use various types of convergences of sequences of functions mentioned above. If time permits, we will cover some mathematical aspects of the conditional probabilities and the conditional expectations that depend on the Radon-Nikodym theorem.

Prerequisites: Real Analysis MATH 4030, MATH 5010 or equivalent.

Main Topics:

1. Sigma-algebras of subsets. Borel sigma-algebra.
2. Measure. Lebesgue-Stieltjes measures.
3. Measurable functions. Various types of convergence (almost everywhere, almost uniform, in measure).
4. Lebesgue Integration; Monotone and Dominated convergence theorems.
5. Product measures, Fubini's theorem.
6. Measure transformation Theorem
7. Probability interpretation of the Measure Theory concepts (random events, probability, random variables, expectation and higher moments, distribution).
8. Independent sigma-algebras; independent random variables.
9. Zero-One Law.
10. Strong and weak Laws of Large numbers.
11. Characteristic functions of distributions and convergence in distribution.
12. Central Limit Theorem.

Homework and Assessment: Weekly assignments will be posted on the Blackboard, and they will be due on the day of the following meeting (uploaded to the Blackboard). **I will not accept the late homework,** and this may result in a loss of points**. The written work you turn in must be prepared independently** and represent your own understanding of the problem. However, you are encouraged to discuss with me any questions that you may have when working on the assignments. **Your writing and scans should be neat. If I cannot easily read, I will not grade it. Redoing or extra work by my suggestion only.** There will be **in class quizzes** based on the home assignments.

Grading: Final score over 90% translates to A, over 80% to B and over 70% to C.

Academic Integrity: Academic dishonesty is prohibited in all programs of the University and sanctions may be imposed on any student who commits an act of academic dishonesty. Details on UML policy can be found at http://www.uml.edu/Catalog/Undergraduate/Policies/Academic-Integrity.aspx and at http://www.uml.edu/Catalog/Graduate/Policies/Academic-Integrity.aspx . Note in particular that any incident which results in some action being taken must be reported to the Provost's Office.