Modal analysis of a beam structure excited by the ultrasound radiation force

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Abstract. The use of conventional modal excitation techniques (e.g., impact hammers, piezoelectric actuators, and mechanical shakers) may be challenging if high-frequency dynamic measurements on lightweight structures need to be performed. The use of the high-frequency radiation force generated by focused ultrasonic transducers (FUTs) can help to resolve this problem. In this research, the acoustic pressure generated by a FUT is measured using an acoustic microphone and used for determining the force distribution in the focal plane of the FUT by employing a boundary element model (BEM). The results indicate that the radiation pressure field has a spot size of ~3 mm in diameter and the radiation force has a maximum intensity equal to 2.5 10^{-5} N. To validate the capability of the FUT as an excitation technique for modal analysis, a back-to-back comparison with a modal test performed using a traditional excitation system (i.e., shaker) was completed. Analysis of the results when the two excitation techniques are used, shows a good agreement between the data sets. Excellent correlation is also observed when the mode shapes are compared using the Modal Assurance Criterion (MAC). Overall, the good match of both resonant frequencies and mode shapes indicates that the focused ultrasound radiation force excitation technique is helpful in performing modal tests.

1. Introduction

The capability to perform accurate dynamic measurements is essential for a wide range of applications. Conventional modal testing excitation techniques, typically rely on impact hammers, piezoelectric actuators, or mechanical shakers excitation. However, the use of these devices may be challenging if accurate high-frequency dynamic measurements on small or lightweight structural parts have to be performed [1]. Moreover, the physical connection may lead to mass and stiffness loading effects; issues that distort structures' dynamic characteristics (e.g., natural frequencies, mode shapes, and damping) [2]. Therefore, providing mechanical excitation to small or lightweight structures without interfering with their dynamic characteristics to obtain information to estimate Frequency Response Functions (FRFs), natural frequencies, mode shapes, and damping, is still a challenge that needs to be addressed.

In recent years, non-contact excitation methods have been explored as potential approaches for exciting and detecting vibration on structures having a size ranging from the micro- to macro-scale. Researchers performed a considerable amount of work to understand the physics of the acoustic radiation pressure and force. In particular, the work made by Westervelt has to be recognized as one of the first and most influential on this topic [3]. For instance, direct acoustic excitation has been used as

a non-contact method for operational modal analysis in the audio range up to ~40 kHz [4, 5]. Unfortunately, the sound generated by those transducers is not focused, and the resulting acoustic force is imparted over a distributed area, preventing accurate estimation of FRFs. By employing transducers emitting sounds at a higher frequency, the dimension of the area over which the excitation is applied can be reduced to a few millimeters, and the radiated sound may be used as an effective non-contact method for structures such as micro-cantilevers and small turbine blades.

The steady acoustic radiation force has been used in a variety of applications, including the measurement of sound intensity and power out of Ultrasonic Transducers (UTs) [6], acoustic manipulation of microparticles [7], and acoustic levitation [8]. In addition to that, the UT-generated acoustic pressure has been widely employed in non-destructive testing applications, and it is one of the most used techniques in the field of imaging for biomedical applications [9, 10]. Some efforts about the possibility of using ultrasound radiated force as structural excitation technique have also been made for measuring the frequencies and operating deflection shapes (ODSs) [11 – 13].

To perform experimental modal testing, the input force needs to be known or estimated to obtain the FRF (i.e., output/input response) of the system being tested and identify critical modal parameters. To date, the inability to quantify the acoustic radiation force prevents this approach from being used as a practical technique for experimental modal testing and motivates this research. In this study, the dynamic focused ultrasound pressure generated by focused ultrasonic transducers (FUT) is quantified both experimentally and analytically by using a boundary element method (BEM) model based on the Rayleigh Integral. Experiments are used for validating the performance of the BEM in the lower frequency range (i.e., 50 - 80 kHz). Once validated, the model is used for predicting the behavior of the FUT at higher frequencies (i.e., above 300 kHz) and the computed pressure fields are used to calculate the acoustic radiation force generated by the transducer. Finally, the capability of the FUT as an excitation technique for modal analysis is validated. The FUT is used for exciting a cantilever beam during a modal test, and results are compared with those recorded as the same structure is excited using a shaker.

2. Theoretical background

Commercially available microphones can measure acoustic pressure fields up to 100 kHz. Therefore, a model is the only possible way to determine the pressure field first, and the acoustic force radiated by a FUT operating at frequencies higher than 300 kHz.

The FUT used in this research is a single element amplitude modulated (AM) device that can be driven by a double sideband suppressed carrier (DSB-SC) AM signal to generate sinusoidal signals that can be used to generate a harmonically varying force [14]. In this configuration, as two ultrasound beams with frequencies f_1 and $f_2 = f_1 + \Delta f$ are superimposed, the constructive and destructive interferences between the beams produce a radiation force having a vibration frequency equal to the difference frequency $\Delta f = f_2 - f_1$. The total pressure field $p(\boldsymbol{r}, t)$, generated by the two frequency components in a point distant $\boldsymbol{r}$ from the FUT can be evaluated using Eq. (1):

$$p(\boldsymbol{r},t) = P(\boldsymbol{r})\cos[2\pi f_1 t + \varphi_1(\boldsymbol{r})] + P(\boldsymbol{r})\cos[2\pi f_2 t + \varphi_2(\boldsymbol{r})] \quad (1)$$

If the two waves have difference frequency Δf far less than both f_1 and f_2; p_1 and p_2 can be considered to be almost the same and equal to $P^2(\boldsymbol{r})$ with phases $\varphi_1(\boldsymbol{r})$ and $\varphi_2(\boldsymbol{r})$ respectively [3]. Therefore, under the assumption that the waves are planar and propagate with speed c in a fluid having density ρ, the ultrasound beams will have an energy density $E(\boldsymbol{r}, t)$ governed by Eq. (2):

$$E(\boldsymbol{r},t) = \frac{p^2(\boldsymbol{r},t)}{\rho c^2} = \frac{P^2(\boldsymbol{r})\cos[2\pi\Delta f t + \Delta\varphi(\boldsymbol{r})]}{\rho c^2} \quad (2)$$

The total radiation force in time-varying form $F_{\Delta f}(\boldsymbol{r},t)$ over the surface area of the structure impacted by the incident sound waves can be calculated using the Eq. (3) for message frequencies much smaller than the carrier frequencies (i.e., < 1%) [15]:

$$F_{\Delta f}(\boldsymbol{r},t) = \int_S E(\boldsymbol{r},t)\,\mathrm{d_r}(\boldsymbol{r})\,\mathrm{dS} = \mathrm{d_r}(\boldsymbol{r})\int_S \frac{(p_1+p_2)^2}{\rho c^2}\,\mathrm{dS} =$$

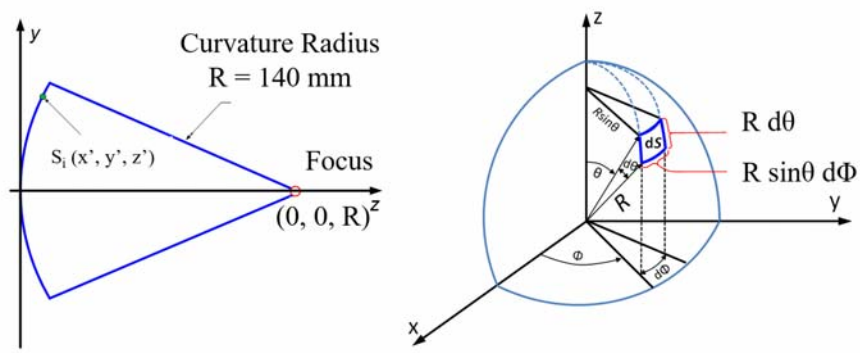
$$= \mathrm{d_r}(\boldsymbol{r})\int_S \frac{P^2(\boldsymbol{r})}{\rho c^2}\cos[2\pi\Delta f t + \Delta\varphi(\boldsymbol{r})]\,\mathrm{dS} \quad (3)$$

where $\boldsymbol{d_r}$ is the drag coefficient vector as defined in [3], dS the scattered area, and p_1 and p_2 the pressure of the two incident planar waves considered. It should be noted that the resulting acoustic radiation force is in proportion to $(p_1+p_2)^2$, which include a DC component, a summation component (f_1+f_2), a doubling of the frequency component ($2f_1$ or $2f_2$), and a difference frequency component (f_1-f_2). As a result of the interaction of a test object with the ultrasound radiation force, the structure will be vibrated at the difference frequency Δf. This quantity is relevant to this research, while the other three components will induce vibrations in the structure that are out of its range of interest or that have negligible magnitude. Therefore, by adjusting Δf, different structural excitation frequencies over the desired bandwidth are achievable as this technique is employed. It implies that as the transducer is operated with a modulation frequency much smaller than the carrier frequency, the generated wave can be considered a single-frequency plane wave and used as a steady-state source of excitation.

For a focal transducer, the sound pressure of a field point $p_i\ (x, y, z)$ produced by a source element S_i can be calculated using Eq. (4):

$$p_i\,(x,y,z) = \frac{j\omega\rho U\,(x',y',z')\,e^{-jkr}}{2\pi r}\,\mathrm{dS} \quad (4)$$

where r is the distance between the field point and the center of the source element, ω is the circular frequency of the vibration of the transducer surface, ρ is the fluid density, $U\ (x', y', z')$ is the amplitude of vibration velocity of the source element (which is going to be experimentally measured at its geometrical center), dS is the area of the *i-th* source element, and k is the wave number. In this study, with reference to Figure 1, and to take into account the curvature of the transducer, the geometrical terms of Eq. (4) can be calculated as:



(a) (b)

Figure 1 – Diagram of the coordinate system and BEM used to simulate the acoustic field generated by spherically curved ultrasonic transducers. (a) Section of the curved transducer, (b) an individual source element dS that represents a small area of the transducer.

$$\mathrm{dS} = R\sin(\theta)\mathrm{d}\Phi \cdot R\mathrm{d}\theta \quad (5)$$

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \quad (6)$$

where R is the curvature radius of the FUT and the terms x', y', z', in Eq. (6) can be calculated like $x' = R\sin(\vartheta)\cos(\Phi)$, $y' = R\sin(\vartheta)\sin(\Phi)$, and $z' = R - R\cos(\Phi)$ respectively. The pressure calculation method shown in Eq. (4) is derived for simulating pressure field of a plane piston. However, it can be

considered accurate for slightly curved transducers (i.e., almost plane, which is the case analyzed in this study) as well. The overall pressure at a field point is the superposition of pressure contributed from all of the *N* source elements and can be formulated using Eq. (7):

$$P\,(x, y, z) = \sum_{i=1}^{N} p_i\,(x, y, z) \tag{7}$$

with N = 15000 in the case study here described. Eq. (7) is used to generate an acoustic radiation pressure profile for the FUT over numerous points representing an area of interest. The accuracy of the simulation is validated comparing the numerical results with those obtained as the radiated pressure is experimentally measured using an acoustic microphone in the lower frequency range and will help to understand the FUT's radiation characteristics (e.g., pressure, directivity, and spot size) in the higher frequency range.

3. Pressure field characterization

The BEM described in the previous paragraph has been validated through a set of laboratory experiments. In this research, a noncontact circular FUT (model NCG500-D50-P150 from the Ultran Group), with an actual focal length of ~140 mm, has been used. It has a nominal diameter of 50 mm and a nominal operating frequency of 500 kHz [16].

By using a scanning laser Doppler vibrometer (SLDV) (model PSV-400 manufactured by Polytec, Inc.), the velocity profiles *U (x', y', z')* of the transducer surface were measured at discrete frequencies (i.e., 50, 60, 70, and 80 kHz) and entered in Eq. (7) to obtain the pressure distribution of a field point. As shown in Figure 2, a pressure field of 100 × 100 × 450 mm with a spatial resolution of 5 mm was simulated in this research.

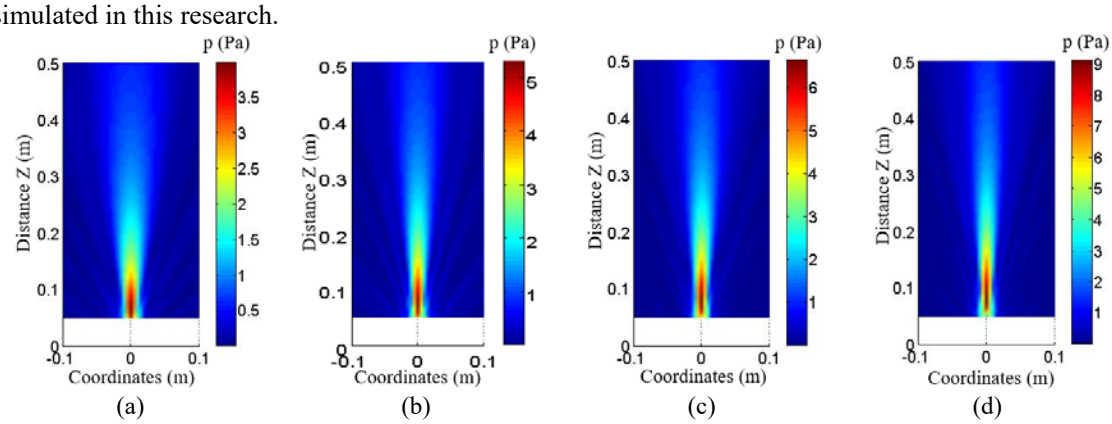
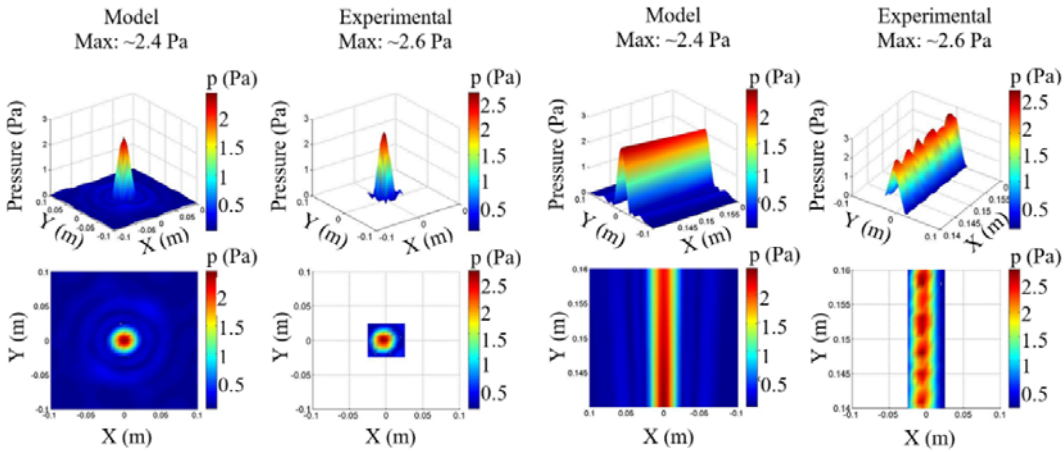


Figure 2 – Predicted pressure fields computed using the BEM in conjunction with the transducer's laser vibrometer velocity measurements for different frequencies: (a) 50 kHz, (b) 60 kHz, (c) 70 kHz, and (d) 80 kHz.

A focal region can be identified. Also, it is possible to notice that as the excitation frequency increases, the focal region gets smaller and the pressure magnitude becomes larger ranging from 3.7 Pa for the 50 kHz excitation to 9 Pa for the 80 kHz case.

To validate the accuracy of the BEM, a back-to-back comparison was performed with the pressure measured using a ¼″ acoustic microphone Model 378C01 manufactured by PCB Piezotronic, Inc. Measurements were made in the three planes P_1, P_2, and P_3, parallel to the surface of the FUT, and in one horizontal plane P_4, perpendicular to the surface of the FUT. P_1, P_2, and P_3 were 140, 150, and 160 mm away from the FUT surface, while plane P_4 extended in a region spaced 140 and 160 mm from the FUT. Figure 3 shows an example of the pressure fields measured using the microphone and those simulated using the BEM. For the sake of brevity, only results in plane P_1 and P_4 for a driven frequency of 50 kHz are shown here. Experimental data and numerical simulations are in excellent agreement. The comparison indicates that the pressure values are close and the pressure profiles are similar.



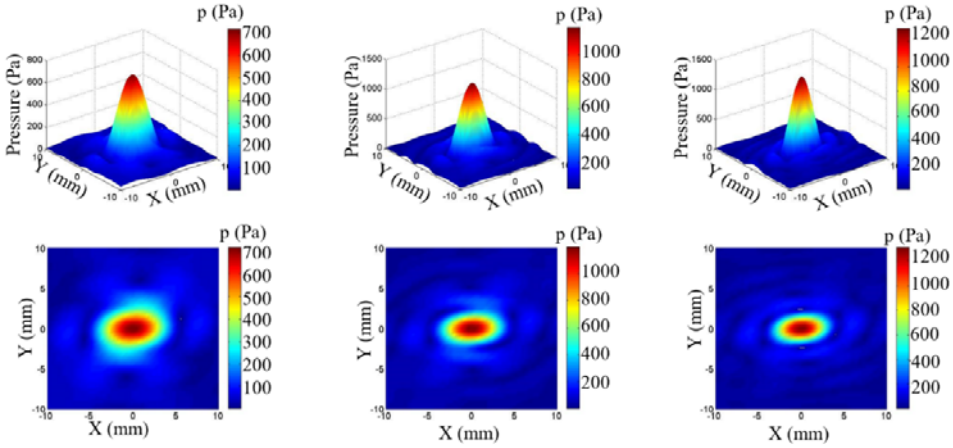
(a) (b)

Figure 3 – Comparison of model and experimental ultrasound pressure when the FUT is excited at ~25 V_{rms} and 50 kHz for two measurement plane slices: (a) P_1 = 140 mm, (b) P_4 = 140 - 160 mm.

The simulation results seem to underestimate the actual value of the maximum acoustic pressure value, but the average error for the four planes considered is equal to 0.15 Pa and can be considered relatively small.

3.1. FUT pressure field simulation in the higher frequency range

With the availability of the validated acoustic BEM model, the ultrasound pressure field in the higher frequency range (i.e., the carrier frequency range of the FUT) can be predicted. Vibrational FRF testing using an SLDV was conducted to determine the velocity profile *U* of the FUT in the 300 - 400 kHz frequency range. The full ultrasound pressure field at three different frequencies (i.e., 325, 362.5, and 400 kHz) for the plane P_1 is shown in Figure 4.



(a) (b) (c)

Figure 4 – Simulated ultrasound pressure field in the plane P_1 = 140 mm at (a) 325, (b) 362.5, and (c) 400 kHz.

Compared with the results at lower frequencies, the pressure value at carrier frequency level is approximately three orders of magnitude higher. The maximum value of the ultrasound pressure increases as frequency increases reaching peaks higher than 1200 Pa as the 400 kHz excitation is considered. Moreover, it is possible to observe that a highly focused acoustic spot with a diameter of ~3 mm is produced.

3.2. Force quantification

Once the pressure field is known, the ultrasound radiation force $F_{\Delta f}(\mathbf{r}, t)$ as defined by Eq. (3), can be estimated for a given carrier frequency. The distribution of the predicted ultrasound radiation force for 325, 362.5, and 400 kHz is shown in Figure 5 under the assumption of a perfect reflection of sound waves by test article (i.e., drag coefficient $\mathbf{d_r}$ sets equal to 2).

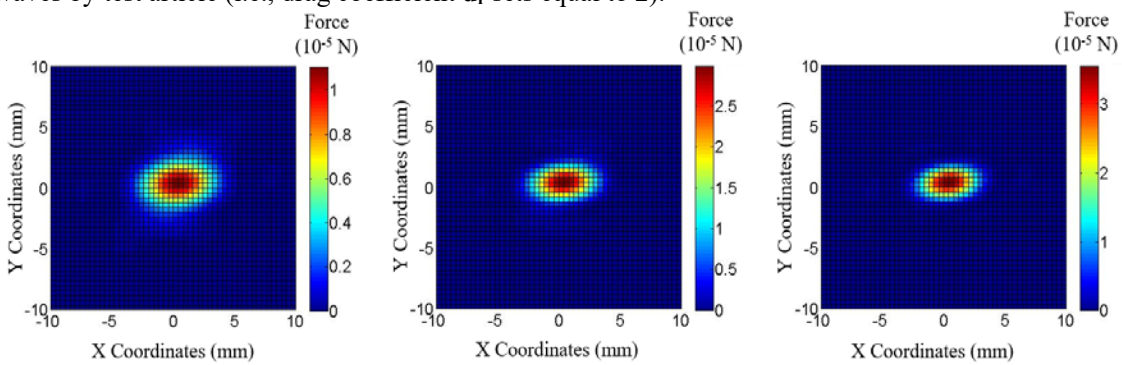


Figure 5 – Quantification of ultrasound radiation force $\mathbf{F}_{\Delta f}(\mathbf{r}, t)$ acting on a test article140 mm away from the FUT for three frequencies: (a) 325 kHz, (b) 362.5 kHz, and (c) 400 kHz.

It can be found that the majority of ultrasound radiation force is confined within a range of ~3 mm in diameter around the geometrical center of the FUT and that it has maximum values of ~1.3×10^{-5} N, 2.7×10^{-5} N, and 3.5×10^{-5} N for the three frequencies considered.

4. Experimental modal analysis

To validate the capability of the FUT as an excitation technique for modal analysis, a back-to-back comparison with a modal test performed using a traditional excitation system (i.e., shaker) was completed. The structural response of the upright structure shown in Figure 6 was measured using a Polytec SLDV as the beam structure was excited using an electromechanical shaker driven with a pseudo random signal from 0 Hz to 4 kHz applied at the center of the mounting base, and with the FUT set to have a carrier frequency of 359 kHz with a frequency modulation of 20 Hz.

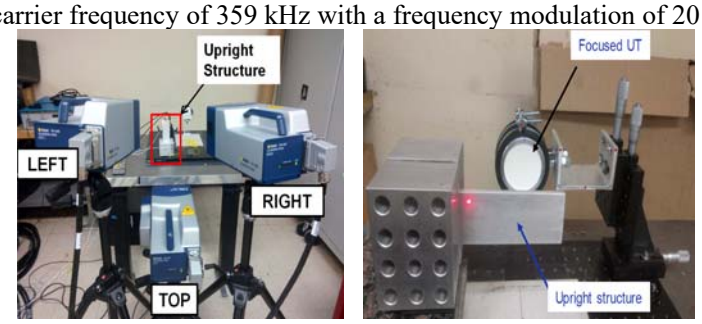


Figure 6 – Experimental setup of modal testing and SLDV for displacement measurements: (a) using shaker excitation; (b) using ultrasound radiation force excitation.

The results of the modal test performed as the shaker excitation is used are shown in Figure 7 and Table 1, where the values of the first four natural frequencies for the test object are summarized. In this test, 45 scan points were used as measurement pattern by the SLDV set-up in a three-dimensional configuration. Using the modal information, the dynamic characteristics of the target structure can be determined. It can be seen that the first flexible mode of the upright plate structure is its first bending mode at ~371 Hz, the second flexible mode is its first torsional mode at ~1585 Hz, the third flexible mode is its second bending mode at ~1966 Hz, and the fourth flexible mode is its first in-plane

bending mode at ~2499 Hz. In this study, the rigid body modes in low-frequency range are neglected and only the flexible modes are assessed.

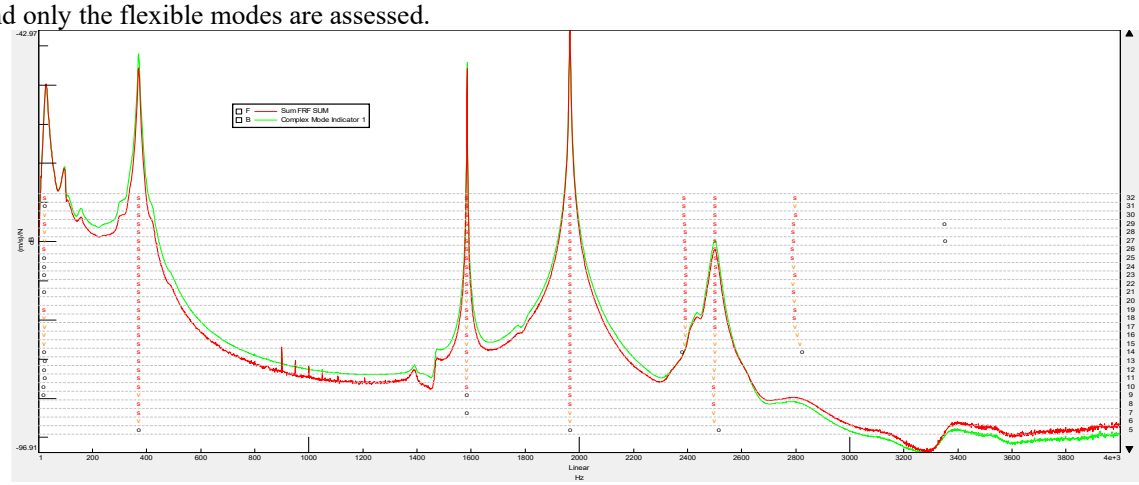


Figure 7 – Stabilization diagram obtained as the upright structure is excited using the shaker.

Table 1: Natural frequencies determined from modal analysis using the shaker.

Mode number	Mode name	Frequency
(-)	(-)	(Hz)
1	1st bending	371.4
2	1st torsional	1584.5
3	2nd bending	1965.8
4	1st in-plane bending	2498.9

After that, the FUT was used to excite the first three flexible modes of the structure, the FRFs were measured, and the modal parameters were extracted. In this testing, a DSB-SC AM signal with a carrier frequency of 359 kHz with a random variation of 10 cycles at 10 kHz was used as an excitation signal. To avoid damage to the FUT, the bandwidth was limited to 20 Hz to prevent it from getting hot. Due to the defocusing effect, only the first three flexible modes were tested. Therefore, based on a previously developed finite element model of the structure, the excitation frequencies were defined as 360-380 Hz, 1570-1590 Hz, and 1970-1990 Hz. Two-dimensional vibration measurements were made using the SLDV with a frequency resolution of 1 Hz, and only 21 points were scanned to limit the usage cycle causing heating of the transducer. The identified modes and natural frequencies are shown in Table 2.

Table 2: Natural frequencies determined from modal analysis using the FUT.

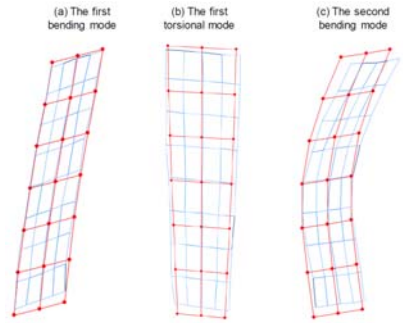
Mode number	Mode name	Frequency
(-)	(-)	(Hz)
1	1st bending	371.1
2	1st twisting	1579.3
3	2nd bending	1980.6

It can be seen that first mode is the first bending mode at 371.1 Hz, the second mode is the first twisting mode at 1579.3 Hz, and the third mode is the second bending mode at 1980.6 Hz.

Excellent correlation is also observed when the mode shapes are compared using the Modal Assurance Criterion (MAC). From data reported in Figure 8, it is possible to observe that MAC values for the first three flexible modes, are equal to 98.9%, 94.9%, and 84.2% respectively. The good MAC values imply a good correlation of mode shapes between the shaker testing and ultrasound radiation force excitation testing. The correlated mode shapes for the first bending mode, first torsional mode, and second bending mode are also presented in Figure 8b.

MAC (%)		Test Mode - Shaker		
		1	2	3
Test Mode - Ultrasound	1	98.9	0.2	7.1
	2	0	94.9	.0.0
	3	0.1	0	84.2

(a)



(b)

Figure 8 – Analysis of the results: (a) MAC values; (b) mode shapes correlation between the shaker testing and ultrasound radiation force excitation.

Overall, the excellent correlation of both resonant frequencies and mode shapes indicates that the focused ultrasound radiation force excitation technique is helpful in performing modal tests. Differences in the MAC at the higher frequency modes may be attributed to the coupling and influence that the attached shaker has on the structure.

5. Conclusions

In this study, the acoustic pressure field and the acoustic force generated by the focused ultrasonic transducer are determined and simulated using a boundary element model for carrier frequencies higher than 300 kHz. The results indicate that the radiation pressure field has a spot size of ~3 mm in diameter and the radiation force has a maximum intensity equal to 2.5 10^{-5} N.

The effectiveness of the dynamic ultrasound radiation force as an excitation technique was analysed. Two different modal analysis methods, shaker excitation, and dynamic ultrasound radiation force excitation are presented to perform modal analysis on a target beam structure. Structural dynamic characteristics including resonant frequencies and mode shapes are compared and correlated. The percent difference in resonant frequencies is less than 2%, while MAC values are reasonably high. The results demonstrate that both the resonant frequencies and mode shapes are in good agreement. Overall, the excellent correlation indicates that the dynamic focused ultrasound radiation force excitation is a valid and useful non-contact excitation option in performing modal tests for very small or lightweight structures.

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